Received: 4 April 2024 | Accepted: 9 April 2024

DOI: 10.1111/2041-210X.14335

Methods in Ecology and Evolution — BRITISH ECOLOGICAL SOCIETY

FORUM

Response to Discretising and validating Keyfitz' entropy for any demographic classification

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Funding information
Natural Environment Research Council, Grant/Award Number: NE/X013766/1; Schweizerischer Nationalfonds zur Förderung der Wissenschaftlichen Forschung, Grant/Award Number: P500PB_211003

Abstract

1. In recent years, demographers have parsed variation in survivorship into two distinct components: the shape and the pace of ageing. The pace of ageing is defined by measures of longevity such as mean life expectancy or maximum longevity, whereas measures of the shape of ageing attempt to classify different shapes of the survivorship curve. We recently published a paper pointing out that a commonly used discretization of a shape measure, Keyfitz' entropy, does not correctly classify survivorship curves into negatively and positively senescing curves (de Vries et al., 2023). In that paper, we also suggested an alternative, accurate discrete-time version of Keyfitz' entropy. de Vries et al. (2023) ended with two open questions, both of which have been answered by Giaimo (2024) now: (1) Can a discrete-time entropy measure of survivorship be generalized beyond age-based population models? Giaimo (2024) introduce a new formula that achieves this. (2) Will a discretization of Keyfitz' derivation of his measure as the elasticity of lifespan to a uniform change in mortality lead to the same formula as a discretization of Keyfitz' result? Giaimo (2024) answers: no.
2. Here, we briefly discuss the implications of the results obtained by Giaimo (2024), and the implementation of his new formula into the Rpackage Rage, which is widely used for comparative demographic studies.
3. We showcase the strength of the new method by reanalysing a comparison of Keyfitz' entropy to another shape measure. We find that the comparison is significantly altered by using Giaimo's new Keyfitz' formula.
4. This example strengthens Giaimo's (2004) words of warning in approaching discretizations with attentiveness.

KEYWORDS
demography, Keyfitz' entropy, mortality, senescence, shape of life

Supported by the emergence of multiple demographic datasets (e.g. Gaillard et al., 2021; Levin et al., 2022; Salguero-Gómez et al., 2015, 2016) and long-term studies (e.g. Clutton-Brock & Pemberton, 2004; Festa-Bianchet et al., 2017; Nussey et al., 2013; Pemberton et al., 2022), ageing research and comparative demography have undusted (Colchero et al., 2016; Gaillard & Lemaître, 2020; Wrycza et al., 2015), and developed (Baudisch & Stott, 2019) new metrics to quantify actuarial senescence—the fast decline in survival with age. Our recent article, *Discretising Keyfitz' entropy for studies of actuarial senescence and comparative demography* (de Vries et al., 2023) contributes to this momentum by introducing a new formulation of Keyfitz' entropy for discrete time, age-based models. In it, we ended with two open questions: (1) Can a discrete-time entropy measure of survivorship be generalized beyond age-based population models? Giaimo (2024) introduces a new formula that achieves this goal. (2) Will a discretization of Keyfitz' derivation of his measure as the elasticity of lifespan to a uniform change in mortality lead to the same formula as a discretization of Keyfitz' result? Giaimo (2024) also delivers an answer: no.

In our article, we pointed out problems with the existing discrete-time formula for Keyfitz entropy, and we suggested a remedy which was limited to age-specific models. Giaimo (2024) introduces a general discrete-time formula that specifies the relevant scalar for any stage-based matrix population models, which of course also works for age-based models. Stage-based population models accommodate vital rates that characterize more complex life histories, such as stasis, multi-class, fast growth/development, and shrinkage (Salguero-Gómez & Casper, 2010), common features in organisms such as plants (Csergő et al., 2017; Gross et al., 2006), corals (Cant et al., 2021; Riegl & Purkis, 2015) but also many mobile animals (e.g. reproduction sabbatical and its return in some birds and marine mammals; Sanz-Aguilar et al., 2011; Weimerskirch et al., 2015). By providing a general form of the correct measure for survivorship changes through time, Giaimo (2024) extends application of the Keyfitz'-entropy analogue to virtually any structured population model. This development has wide practical implications for the field of research on senescence and we are excited by this step forward and thank the authors for answering our open questions. To encourage use of the new Keyfitz formulas, we have renamed 'entropy_k' to 'life_elas' in our Rage R package (Jones et al., 2021), and added discrete time, age-based formulation (de Vries et al., 2023) and the new stage-based formulation (Giaimo, 2024) in the same package as 'entropy_k_age' and 'entropy_k_stage'.

The subtleties of Keyfitz' entropy clarified by the authors are emblematic of broader imprecisions that persist in demographic biometrics (Griffith et al., 2016). Several competing shape measures persist in the literature and, in some cases, conflict with one another. For instance, Healy et al. (2023) recently criticized the role of Keyfitz' entropy as a valid measure of survivorship convexity. In their paper, they use a 'shape of mortality' metric (a variation on the 'shape of fecundity' metric (Baudisch & Stott, 2019)) to show that the existing Keyfitz' entropy metric fails to capture early-life features of survivorship. Since they used the biased Keyfitz' entropy measure that was in circulation at the time, we endeavoured to briefly update the comparison here. A new set of conflicts occurs between the correct Keyfitz' entropy metric and their shape metric that may warrant further investigation (visually shown in Figure 1). Further work on this comparison will be forthcoming.

Throughout their paper, Giaimo (2024) highlights the importance of discrepancies when estimating the same property in continuous time and discrete time. We agree and think that this is a caution that remains relevant now as ever. This admonition is redolent of a

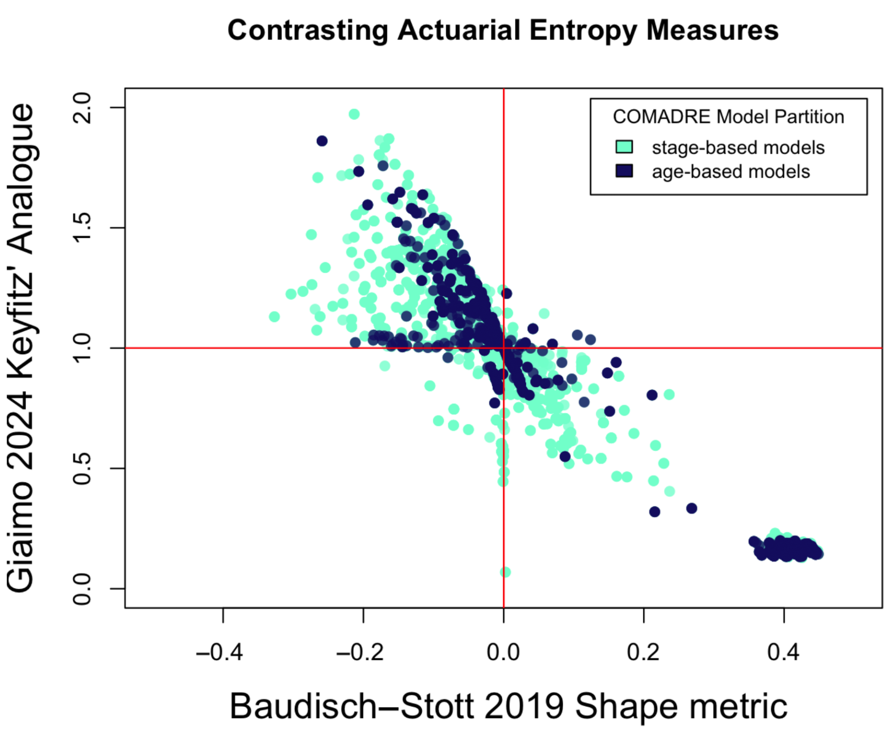

FIGURE 1 Comparison of the Giaimo (2024) updated form of Keyfitz' entropy's survival metric and Baudisch and Stott (2019) shape of survival metric for models in the Comadre database. The thresholds for survival convexity corresponding with increasing or decreasing survival with age are 1 and 0 respectively (both shown in red). Colours correspond with age- (dark blue) and stage- (light teal) based models.

message in Robert May's seminal 1976 paper stressing the complexity innate to a remarkably simple equation: the discrete logistic. By extension, May highlighted how complexity is lost by simply using a continuous analogue of the same equation. We second Giaimo's (2004) words of warning in approaching any time discretizations with attentiveness as we move forward in a time of growing data for continuous and discrete time population models. We again thank the author for generalizing our findings.

AUTHOR CONTRIBUTIONS

Roberto Salguero-Gómez, Connor Bernard and Charlotte de Vries conceptualized the article; Roberto Salguero-Gómez proposed the structure; Connor Bernard wrote the first draft and performed analyses in R; and all authors contributed to subsequent drafts. Owen R. Jones and Roberto Salguero-Gómez implemented changes in Rage.

ACKNOWLEDGEMENTS

Roberto Salguero-Gómez was supported by a NERC Pushing the Frontiers Grant (NE/X013766/1). Charlotte de Vries was supported by the Swiss National Science Foundation grant P500PB_211003.

CONFLICT OF INTEREST STATEMENT

The author declares no conflict of interest.

PEER REVIEW

The peer review history for this article is available at https://www.webofscience.com/api/gateway/wos/peer-review/10.1111/2041-210X.14335.

DATA AVAILABILITY STATEMENT

All data are included in the manuscript.

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How to cite this article: Bernard, C., de Vries, C., Jones, O. R., & Salguero-Gómez, R. (2024). Response to Discretising and validating Keyfitz' entropy for any demographic classification. *Methods in Ecology and Evolution*, *15*, 1286–1289. https://doi.org/10.1111/2041-210X.14335