Symposium on Teaching and Learning Quantum Physics

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Abstract The advancement of quantum technology is putting in the spotlight the question of quantum awareness or quantum literacy among the general population. Therefore, initiatives have been put forward to bring quantum mechanics to pre-university level. Some countries entered quantum mechanics in their regular high school curriculum, and some did not. Various approaches have been developed with varying success. The symposium brought together experts with experience teaching quantum mechanics at high school and introductory university levels. Following the logic of creating a curriculum, various considerations have been identified that play a role in what approach a particular instructor chooses. A main difference emerged between making connections with classical physics using potential wells and barriers as context versus breaking up with classical physics using quantum technology as context. The article presents a synthesis of the discussion on what to teach, how to teach, how to choose between different approaches and how to prepare teachers.

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1 Introduction

Quantum physics and its implications for future science and technology is becoming culturally important and more attention is being given to its introduction before university level [1, 2].

There is rich literature about different approaches and strategies [3]: (1) historico-philosophical, (2) matter-wave, (3) two-state systems, (4) Feynman path integrals, (5) quantum field theory, (6) quantum technology, which have been all adapted for pre-university level. Further differences are in: (a) contextual aspects: e.g. choosing a spin, polarization or double well two-state system, and (b) methodological aspects, using frontal traditional presentation or active engagement methods. All approaches show learning gains among students.

Given the diversity of approaches, there are growing efforts [4] to identify key concepts to be taught and key competences to be learned hinting that all approaches might not be equally suitable to introduce all concepts. To address these issues, a symposium has been organized. The symposium has been structured to follow the construction of a curriculum, starting from the expected prior knowledge, passing through teaching methods, pedagogical considerations and available tools to arrive at an example of an implementation including teacher training. Four participants and a discussant have been invited in base of their expertise in the field. The key questions to answer were:

- What are the key concepts that students should learn in quantum mechanics courses designed for non-physics majors?
- What tools and strategies help in achieving the learning goals?
- What are the challenges and opportunities of teaching quantum mechanics to high school students and non-physics majors?
- What are the considerations taken into account when choosing a particular approach to teaching and learning quantum mechanics?
- How to prepare teachers to teach the topic?

It emerged from the symposium that several of the answers are dependent on a set of considerations about the goals and structure of the course. In their contributions, each participant addressed several of the above topics as they relate to their particular experience. From this, a synthesis emerged that is described in this article.

The article starts with theoretical frameworks about teaching and learning so that various suggestions and examples from participants can be adequately framed in the context of these frameworks. Then the answers to the questions posed above are discussed, as they emerged from the presentations and the following discussion. In the end, a discussion is presented about the potential open questions for further research.

2 Theoretical Frameworks

The symposium revealed that there are multiple theoretical frameworks being used when assessing the educational strategies regarding quantum mechanics. Different aspects of teaching and learning require consideration of different frameworks of teaching and learning.

Starting from the introduction of the completely new physical laws encountered in quantum mechanics it is important to consider student prior knowledge. The *Knowledge in pieces* framework derived from the works of diSessa [5] and Hammer [6, 7] interprets knowledge as being constructed from small pieces called reasoning *resources*. These are rules that are applicable to a variety of situations, such as peaks of waves add up, but are often not applicable universally. According to this framework, many of student difficulties arise from inappropriate activation of otherwise useful resources. An example from mechanics is the "force is proportional to velocity" resource. This resource is adequate in cases where drag is involved. And students' experience of these cases is vast (cycling, cars, dragging objects through water or even pushing a cart to some degree). It is no surprise then that students activate this resource also in cases without friction and drag. In fact, they have much less experience with such examples, so Newtonian mechanics where force is proportional to a change in velocity is much more foreign to them. An important element in the activation of the resources are triggers. If the student learns to trigger the "force is proportional to velocity" resource in cases with drag, but activate the Newtonian mechanics resource in cases without drag, they will be more successful. In this framework, the goal of physics is to provide as universal resources as possible and for students to adopt them as much as possible. With proper application, Newtonian mechanics works in cases with drag as well as cases without drag. Another source of student difficulties according to the Knowledge in pieces framework are inappropriately constructed resources. According to the theory, resources are constructed from experience. Students gradually learn the rules and contexts and construct a sort of network of triggers and rules that gets activated as a single resource. This new resource can be constructed from older resources or completely anew. But if students are not exposed to enough different situations where the resource applies, their resource might be incomplete. The peaks add up resource is applicable to explain the interference pattern, but when asked to explain the shape of a single pulse superimposed on another pulse, students often sum up only the peaks, not every point of the pulses [8]. This is likely due to the fact that a resource about summing the pulses point by point was not constructed. Exposing students to these kind of tasks helps them build up more complete resources.

The next step is acceptance of the newly constructed knowledge. According to *Chi's theory of conceptual change*, there are three *ontological categories*: entities, processes and mental states [9]. An important element of conceptual change is the correct classification of new concepts. An example from waves is that students classify a pulse as an entity instead of a process. Then they talk about pulses bouncing off each other or engulfing each other, much like colliding carts [8]. Conceptual change occurs when reclassification occurs. But in quantum mechanics, duality is an example, where classification within the three ontological categories becomes impossible. Either a new category must be created or severe conceptual difficulties are to be expected.

Another challenge in learning quantum mechanics is the acceptance of the theory. It has been observed that students do not have problems learning the rules of quantum mechanics, but do have problems accepting it, because it is counterintuitive to them [10]. According to *Posner's theory of conceptual change* [11], for a concept to be accepted, it has to be understandable, logical and useful. The logical part is most challenging. Since students are associating quantum mechanics with the real world, they expect the theory to be consistent with their classical view. The fact that it is not creates cognitive conflict. Two ways have been proposed to address this conflict. One is to enhance student knowledge of relevant topics in classical physics to help them connect at least parts of quantum mechanics to their prior classical concepts [12]. The other is to start completely anew, treating the quantum world as a *quantumland* with its own laws.

All these frameworks have to be taken into account when designing learning sequences in quantum mechanics. In the following chapters we discuss how they are reflected in the answers to our symposium questions.

3 Methods

The participants of the symposium were selected based on their long standing involvement in the field and their expertise in the topic. They were asked to submit a contribution about what they consider important about teaching and learning quantum mechanics. The symposium questions were identified from their contributions. In the symposium, the participants addressed one or more of the questions in their presentation. Different perspectives on each question have been identified, analysed and summarized by the discussant. In the Results section we address each question separately and attempt a synthesis of the views of the participants.

4 Results

4.1 What to Teach?

The question of what to teach was approached from three sources, one is an overview of European high school quantum curricula [2], another is the consensus achieved in the Quantum Technology community [13] and a third is the consensus reached in the 2019 workshop of the GIREP community on teaching and learning quantum physics [3]. Table 1 shows a summary of the concepts.

Table 1 An overview of important topics to tech as emerged from three sources, an overview of current curricula, a Delphi study with experts in the research field (QT framework) and a consultation between physics education experts (GIREP community). The scores are given in percentages of the total number of surveyed people/curricula in each survey, if given in the source

	Curricula (%) [2]	QT framework (Y/N) [13]	GIREP community (%) [3]
Discrete energy levels (line spectra)	79	No	0
Interactions between light and matter (photoelectric effect)	82	No	0
Wave-like representations	91	Yes	75
Measurement	0[a]	Yes	100[a]
Wave-particle duality	91[a]	No	100[a]
Matter waves (de Broglie)	82	Yes	0
Superposition	0	Yes	75
Technical applications	58	Yes	50
Uncertainty principle	61	Yes	75
Probabilistic predictions	61	Yes	100
Nonlocality	0	Yes	50
Philosophical implications	33	No	75
Nature of science	100	No	50

[a] In Ref. [3] measurement and wave-particle duality have been classified together with the rationale that the particle-like interactions of quantum entities can be considered the consequence of collapse after measurement. In Ref. [2] measurement is not mentioned

As can be seen from Table 1, there is some disagreement about the core concepts that students should learn. This stems from the goals and approaches that the participants envisioned. For example, a wave-like description is irrelevant for a two-state system. Similarly, the Nature of science goals are very important in school curricula, but overlooked at the level of experts, probably assuming that they would be learned in other contexts. One pattern that can be seen from Table 1 is that the wave-like description of matter seems to lose its importance as the goals shift away from a historical and philosophical perspective towards a more pragmatic and contemporary quantum technologies perspective. The choice of topics will be discussed again in the section about the choice of approaches.

In some countries, quantum mechanics is not defined in the core curriculum, in others it only amounts to developments before 1925 [2]. In these countries it can be either taught as an extracurricular activity or in the context of non-compulsory topics within regular classes. In these cases, the choice of topics depends on the teachers and their goals.

Despite some disagreement on what to teach, there is strong consensus on how to teach. An overview of teaching tools and strategies [1] shows that teaching quantum mechanics is generally efficient if done with multiple representations, in an active engagement environment and making the core concepts very clear and accessible. To this end, in the symposium Heusler proposed to focus on models using multiple representations, focus on interpretations with conceptual clarity and technical terminology, focus on conceptual understanding, using mathematics and visualization tools, and focus on activities by looking for accessible experiments and simulations. Bondani further conceptualized the same ideas in the following way: *core concepts* are derived from experiments, and are defined and described with representations, representations are used to analyse and interpret experiments, and successful description or modelling of experiments leads to the definition of *axioms*. In this context, axioms are rules formalized in a mathematical language, that can be used to explain the outcomes of the already observed experiments and predict outcomes of new experiments. They are a formalization of the core concepts.

Krijtenburg-Lewerissa emphasized that sometimes curricula are predefined and in this case the question of what to teach becomes moot [14]. The Dutch curriculum focuses on wave-particle duality, the infinite potential well, and tunnelling. Therefore, it is important that students understand the related physics. Krijtenburg-Lewerissa then presented a research into how students' prior knowledge of potential energy influences their learning of quantum mechanics [14]. Surprisingly, positive effects have been observed already in the pre-test before any instruction on the quantum. The effect of an additional module about potential energy has had small, but significant positive effects on all aspects of learning quantum physics.

From the analysis in Table 1 and the contributions in the symposium, it became clear that instructors generally decide between two very different approaches. Either they start from scratch and treat the quantum world as a quantumland with its own rules still to be discovered [15, 16], or they start from the known, attempting to build bridges between classical and quantum [14, 17].

The quantumland approach allows students to disconnect any of their prior knowledge from the quantum world [18]. This opens the question whether it may be more useful in this case, that the choice of the context is also something students do not know from the classical world, such as spin or polarization, rather than something familiar like position and energy in a double well. This question remains to be answered in future research. When instructors choose this approach, they usually start from various observational experiments. These may include observing light passing through differently oriented polarizers [19], a polarizing Mach–Zehnder interferometer [15, 20], particles with spin passing through a Stern-Gerlach apparatus [21] or simulation of particle detection in a double well [16] to name just a few. The topics are typically indeterminism, superposition, effect of measurement, uncertainty principle and then either towards quantum technologies such as computing and cryptography [15] or towards more derived concepts such as entanglement, "which way" questions and time evolution [16, 19].

The approach connecting classical and quantum allows students to relate their new knowledge to the already acquired knowledge potentially making the new knowledge more relevant and significant. Instructors taking this approach typically focus first on related classical concepts, like waves and potential wells. They then proceed towards adding quantum elements to an already familiar environment in form of wave particle duality and/or statistical interpretation [14]. Krijtenburg-Lewerissa showed that understanding of potential wells can aid in the process. Previous work also shows how other relevant topics can be related to quantum mechanics, such as waves to wave functions [22].

The choice of what to teach is thus very dependent on the goals, but two-state approaches appear to be more aligned with the quantumland approaches emphasizing practicality and current relevance, while matter-wave approaches tend to rely more on the historical development and the philosophical implications.

4.2 *Tools and Approaches*

Based on all cognitive science theories accepting new hypotheses requires experience with them. Posner's theory of conceptual change requires cognitive conflict, while the Knowledge in pieces framework requires experience upon which to build new resources. It is well known that students are perfectly able to learn whatever is taught, but fail to accept it, if not given enough opportunity to test it. Back to the example of Newtonian mechanics, students are capable of learning it, but some still think it only applies in school, because outside of school they have little experience with frictionless dragless environments. In the real world, force is proportional to velocity makes more sense to them. In a similar manner, students need experience with the quantum world to be able to accept it.

Real quantum experiments require specialized equipment that is still beyond the reach of high schools. Nonetheless, there are multiple efforts to make the quantum as accessible as possible. Heusler presented an optical bench made with a 3D printer, which is affordable and designed to enable crucial experiments in optics [23]. While single photon experiments remain generally inaccessible to high school students, the optical kit offers numerous possibilities for analogous experiments, from Michelson interferometer to Mach–Zehnder interferometer. With the addition of polarizing filters an analogy to a quantum eraser can be produced. This paves the way to make the results from single photon experiments (either reported or experienced with a simulation) plausible. Thus fulfilling perhaps the hardest of the requirements from Posner's theory of making the new theory plausible.

For experiments that cannot be affordably done in class or whose results are difficult to interpret, there are multiple simulations available [24–26]. These serve to give the students an impression of inquiry. While simulations may not be particularly persuasive to "confirm" a learned theory, because the theory is obviously already built into the simulation, their potential is greater when exploring new laws of physics. In this context, while students actually investigate the laws of the simulation, they, at the same time, investigate the laws of nature upon which the simulation is built. Students and people in general have no problem accepting new, strange rules for made-up environments, like fantasy worlds or chess, so they may have little difficulty finding out the rules of the simulation. The challenge is persuading them that these rules apply to the actual world, if they appear counterintuitive to them. To address this, learning the nature of science and scientific models may be crucial and will be discussed in the next section.

An important learning tools are representations [27]. They have been addressed by all participants. It is known that using multiple representations helps learning [28]. In quantum mechanics, the already well established Dirac notation and Feynman diagrams immensely help experts to talk about complex concepts. Multiple suggestions have been made over the years about how to simplify the mathematics of quantum mechanics for the benefit of high school students or about introducing completely new notations [16, 19, 29]. However, for high school, a standardized notation has not yet been developed. Nevertheless, in the symposium it emerged that the Dirac notation might be very useful in high school, too, albeit with some modifications. This is supported by the experiences with a summer school, presented by Bondani, where Dirac notation is used [15].

Focusing more on quantum computing, they represent different eigenvalues with “1” and “0”. In addition, they use the representation with quantum gates. This representation has the advantage to clearly show which qubits are being operated upon. It also clearly distinguishes between operations on single qubits and operations on multiple qubits. Bondani showed that participants efficiently use both representations and are capable of using them complementarily.

Heusler proposed a representation involving a Bloch sphere inside a ket. This representation can be used for any two-state system. Examples given in Fig. 1 are from spin and double well, but the representation can also be used for polarization, neutrino oscillations, and mass-flavour eigenstates.

A long used representation is that of the wave function in a potential. This is more natural for the wave function approach discussed by Krijtenburg-Lewerissa. Wave functions can be very dynamic, so visualization tools are very useful in addressing any temporal components of wave functions. Heusler presented the Quantum composer [30], a visual tool used to manipulate potentials and represent wave functions. These

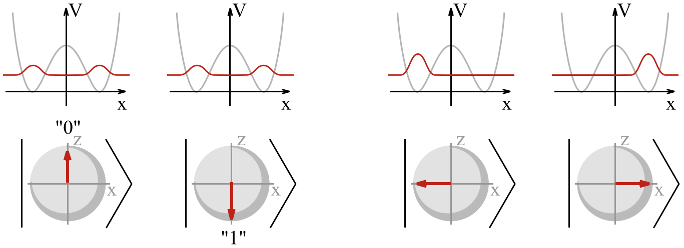

Fig. 1 States represented with Bloch-sphere kets. The figure shows how stationary and non-stationary states in a double well can be represented with Bloch-sphere kets

can be represented in multiple graphical forms, 2D graphs, 2D contour plots, etc. The strength of Quantum composer as compared to similar PhET tools [31] is in its blocks-like structure which visually represents the relations between the various elements of a quantum mechanical problem. Emigh further emphasised the importance of representational fluency in quantum mechanics instruction [32–34]. This was a major consideration in the design of their course which uses sketches to represent experiments and their outcomes, histograms to represent probabilities, Dirac notation, matrix notation and even kinesthetic representations, where students represent phases with their arms (see Fig. 2).

Heusler further proposed a topological representation of states using a belt [35] (see Fig. 3). In this model, a Moebius belt represents one possible outcome of a measurement. The Moebius belt can have either a left or a right twist, representing two possible outcomes on a two-state system. In this model, an entangled state is created by using a regular belt and twisting one part of it for 180 degrees. Spatial separation is represented by squeezing together the twisted part and a portion of the straight part of the belt creating two apparent Moebius belts, one with a left and one with a right twist. One belt is assigned channel A and the other channel B, but the assignment is random creating entanglement. If a measurement is made on channel A, one gets either a left or a right twisted Moebius belt. A measurement done on the other channel would reveal anticorrelation.

It was a consensus among participants that the appropriate notations and representations are crucial to develop conceptual understanding among students. Especially, there appears to be no need to shy away from the Dirac notation, as it can be easily adopted by students of all ages. There is, however, necessity to investigate what type of symbols are best to use in the kets. For quantum computing, “1” and “0” might suffice and are well accepted. For the spin system, a more pictorial representation like a Bloch sphere (or circle) appears natural.

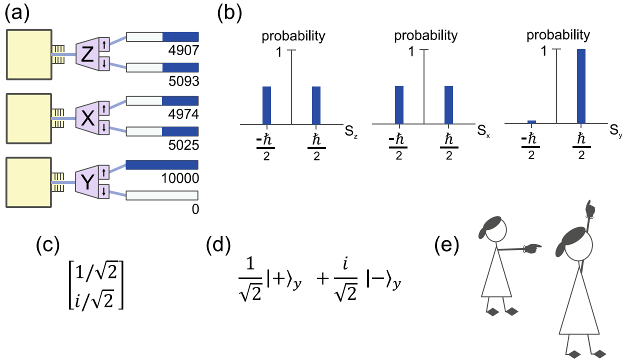

Fig. 2 Various representations used in the paradigms method. **a** sketch, **b** histograms, **c** matrix, **d** Dirac, **e** kinesthetic

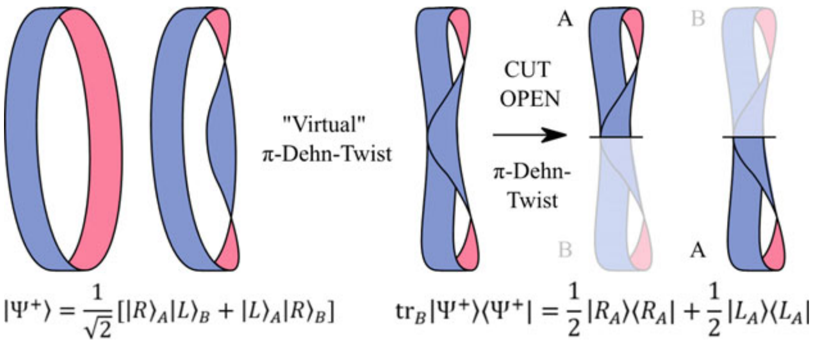

Fig. 3 A topological representation of entanglement. The belt represents entanglement. The twist and squeezing of the belt represent spatial separation. And a measurement is represented by cutting the belt in the middle creating two contrary twisted Mobius belts

4.3 Challenges and Opportunities

While it is known that learning quantum mechanics presents many challenges [36–38], it also presents many opportunities. One is certainly the opportunity to address the nature of science or epistemology of science in a meaningful way.

Looking from multiple perspectives, realizing that classical models do not adequately explain observations in the quantum world is fundamental in the learning of quantum theory. From the perspective of Posner's theory of conceptual change, this creates the necessary cognitive conflict. From more constructivist perspectives, like Knowledge in pieces, it provides the motivation for changing the classical resources of students or building new ones from scratch.

In learning quantum mechanics, there are multiple opportunities to design activities in which students predict the outcome of an experiment based on a classical hypothesis and are then forced to reject the hypothesis based on the outcome of the actual experiment (see for example [16]). These experiments can be done in multiple contexts in very similar ways.

4.4 Choice of Approach

The symposium revealed that the choice of approach is influenced by multiple factors. These factors are tentatively summarized in Table 2. The table shows what elements influence the choice of approach and what considerations are made in the choice as they emerged from the symposium.

Table 2 can help us sort specific approaches in view of the presented considerations. A major factor was observed to be beliefs about efficient conceptual change. The most radical representatives on this dimension appear to be the wave function approach and the quantum technology approach.

Table 2 Considerations that instructors make when choosing an approach as identified in the symposium

Element	Considerations
What do we want students to learn?	Wave function? Two states? Quantum technology? Quantum computing? Should they be able to calculate anything? Measurement? Indeterminism? Uncertainty principle? Nature of science? Philosophical implications? Tunnelling?
External constraints	Is there already an obligatory curriculum? Is the course preparatory for a specific purpose? How much time do we have? Can we change the format of the lectures? Do we have support for experiments or ICT?
Methodological aspects	Do we want lecture format? Do we want an active engagement course? What active engagement framework should we choose? Can students work in groups? Can they have homework?
Theoretical commitments	Do we want to make a connection with classical physics? Do we want to start from scratch? Do we want to build upon existing students' ideas?
Philosophical considerations	Do we want to emphasize the controversies? Do we want to emphasize the apparent paradoxes? Do we want to include the role of the scientific method in philosophy? Do we want to avoid any controversies and apparent paradoxes?

The wave function approach aims at the wave-like representation of particles and the related topics, especially the explanation of the double slit experiment and tunnelling. Useful representations are those of wave functions in potential wells such as provided by PhET [31] and Quantum composer [30]. The Dirac notation is not very suitable for this approach. By its nature, this approach cannot avoid students' prior knowledge (resources) on waves and potentials. Krijtenburg-Lewerissa showed that such approach benefits from deepening the understanding of relevant classical concepts. Among these are also special cases of classical waves [22]. However, depending on philosophical considerations, one may intentionally choose to include controversial and apparently paradoxical topics to incite interest in students, especially students who do not aim at becoming natural scientists and are more interested in the broader philosophical implications of the truthiness of quantum mechanics, such as does this tell us anything about fate and determinism. Krijtenburg-Lewerissa implied that this approach might necessitate changes in the teaching of other related topics in the curriculum. If not, additional time spent on these topics would be beneficial.

The quantum technology approach, on the other hand, starts from scratch with completely new concepts of qubits and quantum gates. These are completely new rules that do not present any relation to existing knowledge. If anything, they evoke resources about programming for those who possess them. Students can learn the new rules as they learn the rules of chess. There is no conflict with any existing knowledge. The simple representations with Dirac notation and quantum gates allows them to predict outcomes of a variety of experiments. Despite the "from scratch" approach, students in the post-test were able to respond to fundamental questions about the concepts of quantum mechanics, such as what does knowing a state imply and how would you respond to someone's particular statement on quantum mechanics.

A more nuanced example of how the considerations in Table 2 influence the choice of instructional method was given by Emigh [33]. He compared the *Paradigms* and *Tutorials* approaches to teaching quantum physics. Both courses commit to a social constructivist approach in which students should build their own knowledge for which interactions between students are crucially important. However, there are differences in the considerations about other topics. Emigh emphasized the structural differences (how the course is scheduled and taught) and considerations about students' resources (what is assumed that students would already know).

In terms of structure, the Paradigms approach requires an overhauling of the schedule. Multiple active engagement methods are used and the course is compacted into a few weeks. In terms of students' resources, the Paradigms approach does not assume what prior knowledge students have, instead it asks and responsively adapts to students' ideas. For example, in the Dirac representation of states, each term has a complex coefficient in front of it. Complex coefficients may present a difficult conceptual node. To test students' ideas about them, they are asked to indicate with their left arm the complex phase of the coefficients describing a particular state (see Fig. 2e). Pointing directly in front of them represents phase zero real number), pointing to the ceiling represents phase $\pi/2$ (imaginary number) and so on. The instructor can then observe the students and react to the ideas that emerge as they emerge.

The Tutorials approach can be adopted in a regular course with lectures and recitations, no scheduling changes required. Tutorials are typically used in recitation sessions. In terms of students' resources, tutorials rely on the assumption that some student responses are predictable. These responses are usually derived from open ended questions with pilot cohorts and then used to prompt discussions in the following cohorts. As an example, tutorials contain dialogue between fictional characters about a topic wherein ideas that have emerged in the pilot cohorts are expressed and students have to comment on the statements made by the fictional characters. In particular, they are asked to explain why a statement is incorrect. This forces them to explore the reasoning of the fictional character and thus compare and contrast their own reasoning with it. This a powerful learning tool encouraging reflection. This type of problem is not limited to tutorials. Bondani mentioned using dialogue or statements attributed to fictional characters in their pre- and post-tests.

4.5 Teacher Training

If we want to bring quantum to schools, teacher training is fundamental. In some countries, high school physics teachers are taught at physics departments and learn the basics of quantum physics like any other physics student. In some countries they are taught at educational departments. In this case, it may be that they have never encountered quantum physics beyond that which is already in the high school curriculum. In some cases, physics in high school is taught by teachers of biology, chemistry, mathematics or general natural sciences. These have probably not encountered quantum physics ever before.

Bondani presented an interesting approach to teacher training. A summer school was organized taking 4 days for 7 h per day which welcomed students and teachers alike. The course was based on quantum technology. In this course, teachers and students experienced the same learning path.

Emigh did not directly address high school teacher preparation, but did address the preparation of the university teaching team consisting of the instructor and graduate or undergraduate teaching assistants or learning assistants. The preparation for the Paradigms and for the Tutorials consists of having the team solve the same problems as the students do, followed by discussing not only the expected conceptual difficulties, but also the conversations that they as instructors want to have with the students. What would be productive and why. This is very in line with Bondani's idea of having teachers and students go through the same materials. With the addition of pedagogical considerations for teachers at a later stage [39, 40].

While teacher education courses are often linked to other courses at the universities, in-service training programs are more independent and focused on a specific topic. A few common elements emerged from comparison of the programs described by Bondani and Emigh. In both programs, teachers/instructors experience one coherent course in the role of students. They are able to explore the materials that are being used, especially simulations and experiments to gain familiarity with them. Then, they are able to discuss pedagogical considerations, like the reasons for particular activities and their goals with course designers. The next step for teachers, as described in [40] is to design their own course with the support of the program instructors.

5 Discussion

The importance of quantum physics for teaching in schools and universities and for reaching out to the general public, has recently gained a great deal of attention and relevance due to the rapid progress in quantum technologies and in the fundamental understanding of quantum physics.

Many teaching approaches of quantum physics exist, each with very different focus. In the symposium, some characteristics of each approach emerged, which enable us to somewhat classify the approaches along various considerations. Therefore, there is not a unique approach or set of tools to be recommended but rather we present a tentative set of considerations, which are intended to help arrive at the choice of approach and tools based on the answers to these considerations. All these approaches have to be re-examined in the light of increasing knowledge and insights into the nature of the quantum world and learning of quantum mechanics, they have to be modified or evaluated in the light of the new quantum technologies, and taking into account the recently often pursued approach with two state systems.

With regard to quantum technologies, there is a certain focus on attracting professionals who will potentially choose this professional field. But when we think about school, we also have to keep in mind the many students who will pursue other career paths. Therefore, above all, the question arises as to the relationship between general education in quantum physics and a pragmatic, career-oriented approach to quantum technologies. This emerges as a topic for further discussion within the community.

The Symposium highlighted basic considerations to teaching quantum physics. The contributions were selected to follow in some sense the logical path to developing a curriculum, starting with the expected prior knowledge of students, then discussing key concepts and visualisations and instructional considerations. To finish off with an example of a course that includes also teacher preparation.

Krijtenburg-Lewerissa gave the example of the Netherlands and discussed what can happen if quantum physics is newly introduced into the high school curriculum. She emphasises with research that the imprint of classical physics on students affects their learning of quantum physics. The very precise question and survey with an ecologically valid intervention gives insight that a coherence of classical and quantum curriculum might be helpful. The question is whether the gap can be reduced without blurring the difference. There are fundamental differences between the classical and quantum worlds. If we successfully reduce the gap between them, what should be the fundamental differences that remain and how should we best address them? The comparison and contrast between the approaches described by Krijtenburg-Lewerissa and Bondani raises the question of how to achieve coherence in a school curriculum. Should the transition between classical and quantum physics be smooth or should it be a disruption? What aligns better with the general goals of physics education as a whole?

Heusler focused on how to teach key concepts of quantum physics, especially superposition and entanglement, by visualisations of very different types. The very insightful visual aids making the mathematical structure tangible allow deep research and insight into new teaching methods. The Bloch-sphere representation is very versatile and natural for spin states, but it may create difficulties when describing polarization states and double well states. Polarization states may present a particular challenge given that "0" and "1" are represented by up and down on the Bloch sphere, but represent vertical and horizontal polarization in the physical system. Another challenge for the Bloch-sphere representation is how to adapt it for typewriting. Regular fonts do not possess characters representing angles. And drawing a circle for every ket becomes very tedious. All these representations might serve by their variety the overarching goal of using multiple representations in general education in quantum physics, mentioned before. The question is, how to embed these representations in a meaningful manner into a teaching path on the characteristics of quantum physics.

Heusler further argued that the motivation for teaching quantum physics might not be quantum physics itself, but rather the impact it might have on the society. He noted that the transistor had an enormous impact on society, yet we do not teach its operation. Likewise, quantum technologies might have a huge impact on society, but that might not require us to teach the basics of quantum mechanics.

Emigh addressed two concrete approaches to teaching quantum mechanics at introductory university level. It was inspiring to see how different theoretical commitments about learning influence the choice of teaching method. While both adopt active engagement of students, they do it in very different ways. It would be interesting to look more intensely into the learning success under these different conditions.

Bondani presented a summer-school curriculum and an impressive number of activities in the framework of quantum technologies. She argued that the characteristics superposition, entanglement and measurement are to be viewed as a resource to build upon. This could contribute to the demystification of quantum physics. Accordingly, the approach focuses on these concepts introducing the qubit and quantum logic gates and their physical realizations avoiding reference to controversial notions. These represent another set of representations that are particularly useful in the context of quantum technologies but may prove useful in other contexts, too. The sequence with which the line of quantum computing is followed in the outreach activities may represent one possible way into the future of teaching quantum physics. In addition, the activities are designed to include students and teachers alike which is an interesting approach to teacher training. However, it remains to be investigated whether this type of approach would be applicable on a larger scale. The connection with classical physics is still strong among some instructors to the point where some refuse to stop teaching the Bohr atomic model [41].

6 Conclusion

The symposium on teaching and learning quantum physics has been organized to bring together experts on the topic and structured to address an organic set of questions: what to teach, how to teach, what are the challenges and how to train teachers to teach it.

The symposium revealed that the answers to most of these questions depend on a set of considerations. These considerations emerged from the symposium and the discussion and we believe could be a step towards classifying the various approaches and thus helping teachers choose the approach based on their answers to these considerations.

One set of considerations relates to the commitments made by a particular teacher. Among these, one central consideration is: Should we look for coherence with classical physics or should we accept and embrace a complete disconnect? Another consideration is: should we use mathematical expressions, visualizations of wave functions or Dirac notation? This depends on whether we are looking for coherence or not. The wave function provides coherence with waves. Dirac notation is a disconnect since it is not used in classical physics. All participants strongly agree that using multiple representations is beneficial for learning.

A different set of considerations is about external limitations. How much time do we have? Can we restructure the course are we preparing for a particular goal (like an exam or a career in quantum technology)? All participants strongly support active engagement methods, but the specific strategy depends on how we can structure the course. Likewise, the choice of topics to teach depends on whether we are preparing future quantum engineers or educating the general public.

The set of considerations that emerged from the symposium might be a starting point for a more sophisticated set of considerations that would enable teachers to choose an appropriate approach based on their answers to these considerations.

It must never be forgotten that all this is not only about the people who will later work professionally with quantum technologies, but above all about those who are "spectators" but still want to understand these fascinating activities and assess their impact on society. As Wagenschein said, "Understanding the comprehensible that's a human right" [42].

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