

Dynamic determinants of optimal global climate policy[☆]

Michael Grubb^{a,*}, Rutger-Jan Lange^{b,1}, Nicolas Cerkez^{c,1}, Ida Sognnaes^{f,1}, Claudia Wieners^{d,1}, Pablo Salas^{e,g,1}

^a Institute for Sustainable Resources, UCL, United Kingdom

^b Department of Econometrics, Erasmus University Rotterdam, The Netherlands

^c Department of International Development, University of Oxford, United Kingdom

^d Institute for Marine and Atmospheric Research, University of Utrecht, The Netherlands

^e International Finance Corporation, United States of America

^f Center for International Climate Research, Norway

^g Cambridge Centre for Environment, Energy and Natural Resource Governance C-EENRG, University of Cambridge, United Kingdom

ARTICLE INFO

JEL classification:

C61

O30

Q54

Keywords:

Climate Change

Optimal abatement

Induced innovation

Path dependence

Globalcost-benefit

ABSTRACT

We explore the impact of dynamic characteristics of greenhouse-gas emitting systems, such as inertia, induced innovation, and path-dependency, on optimal responses to climate change. Our compact and analytically tractable model, applied with stylized damage assumptions to derive optimal pathways, highlights how simple dynamic parameters affect responses including the optimal current effort and the cost of delay. The conventional cost-benefit result (i.e., an optimal policy with rising marginal costs that reflects discounted climate damages) arises only as a special case in which the dynamic characteristics of emitting systems are assumed to be insignificant. Our analysis highlights and distinguishes from the (often implicit) assumption in many cost-benefit models, which neglect inertia and assume exogenous technology progress. This tends to defer action. More generally, our model yields useful policy insights for the transition to deep decarbonization, showing that enhanced early action may greatly reduce both damages and abatement costs in the long run.

1. Introduction

As concern over climate change grows, objectives to cut greenhouse gas emissions have become increasingly ambitious. The emphasis on more rapid and radical action is reflected in the joint governmental goals agreed in the COP21 agreement in Paris in 2015, and in national targets to reach “net zero” emissions—required if global temperatures are to be stabilized. These now cover all major economies and a large share of global emissions (IPCC, 2022). Such ambition implies major and potentially rapid sectoral transformations, raising important questions about the economics of deep decarbonization, including cost-benefit, optimal effort, and trajectories given the dynamic characteristics of global emitting systems (Nordhaus, 2019; Stern, 2022).

Integrated assessment models (IAMs) of climate change can broadly be divided into stylized aggregate cost-benefit models and more complex process-based IAMs (Weyant, 2017; Nikas et al., 2019). The former (e.g., Nordhaus, 1991, 1993; Golosov et al., 2014; van der Ploeg and Rezai, 2019) are common in the mainstream economics literature, focusing in particular on optimal responses given assumed climate change damages, but often neglect dynamic aspects of the emitting systems such as capital stock and innovation processes (Acemoglu et al., 2012; Vogt-Schilb et al., 2018). The latter type involve more detailed representations of energy and land use systems, including dynamic aspects—as used in the IPCC Assessments (IPCC, 2007, 2014, 2022). Such IAMs, 17 of which are reviewed by Harmsen et al. (2021),

[☆] We thank Simon Dietz and Alex Teytelboym for helpful comments and suggestions. Earlier versions of this paper were presented at the annual conference of the International Association of Energy Economics (IAEE) in Washington, the 6th World Congress of Environmental and Resource Economists (WCERE) in Gothenburg, and the European Association of Energy and Resource Economists (EAERE) in Manchester. We are grateful to participants in these sessions for encouraging and useful feedback.

* Corresponding author.

E-mail addresses: m.grubb@ucl.ac.uk (M. Grubb), lange@ese.eur.nl (R.-J. Lange), nicolas.cerkez@qeh.ox.ac.uk (N. Cerkez), ida.sognnas@cicero.oslo.no (I. Sognnaes), c.e.wieners@uu.nl (C. Wieners), psalabravo@ifc.org (P. Salas).

¹ Funding from the Joseph Rowntree Trust (Salas and Grubb), the ESRC program “Rebuilding Macroeconomics” (Cerkez and Grubb), the ESRC (ES/N013174/1) (Salas), ClimateWorks Foundation (ID# 19-1466) (Grubb and Wieners), and Paul and Michelle Gilding (Prince of Wales Global Sustainability Fellowship in Radical Innovation and Disruption) (Salas) is gratefully acknowledged.

<https://doi.org/10.1016/j.strueco.2024.07.005>

Received 1 November 2023; Received in revised form 31 May 2024; Accepted 22 July 2024

Available online 2 August 2024

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primarily focus on modeling pathways toward fixed goals while drawing on large databases and technology-specific assumptions. The resulting complexity can inhibit transparency, and may not illuminate underlying economic mechanisms or key sensitivities.

This article contributes to a nascent literature seeking to bridge these schools. Building on the intuition in Grubb et al. (1995), we develop a stylized cost-benefit model—both analytically tractable and transparent—to evaluate the optimal balance between emissions-driven changes in temperature (Ricke and Caldeira, 2014; Mattauch et al., 2020) and dynamic features of emitting systems identified in the empirical literature: inertia, induced innovation, and path dependency. Our model allows for an analytic solution that yields insights into how just a few, key, dynamic assumptions affect optimal abatement, with the ultimate goal of informing debates on the optimal effort and timing of abatement, *beyond* assumptions about damages.

First, *inertia* in the system arises most obviously from the duration, construction times, and displacement of long-lived capital stock. The importance of assumptions about the malleability of capital stock in optimal growth models has been known for half a century (Newbery, 1972), but many climate-macroeconomic models which focus on the long run have no inertia, or—implicitly or explicitly—assume a high elasticity of substitution between “green” and “dirty” technologies (e.g., Acemoglu et al., 2012, 2016; Hassler et al., 2020). This becomes problematic in view of the typically long timescales of emitting capital stock (Pottier et al., 2014) and growth rates for clean technology (Wilson and Grubler, 2015). Inertia thus has important implications in the face of higher damage costs (Howard and Sterner, 2017) and in meeting the goals of the Paris Agreement (IPCC, 2022).

This aspect of dynamic costs has recently been represented—at a cost of considerable complexity—in IAMs in terms of capital investment; e.g., in the context of a fixed temperature goal rather than a cost-benefit analysis (Vogt-Schilb et al., 2018), or in a DICE-like cost-benefit model including capital stock (Baldwin et al., 2020). Our treatment is more stylized, parameterizing the scale of such adjustment costs—the resistance to accelerating abatement—in terms of timescales associated with energy system transitions.

Turning to *induced innovation*, it is established that technological progress is induced by investment and scale, including learning-by-doing and economies of scale (see Grubb et al., 2021a, for a systematic review of empirical findings). Induced innovation encompasses endogeneity in innovation between high- and low-carbon technologies (Acemoglu et al., 2012, 2014; Aghion and Jaravel, 2015) and in economic systems more widely (Gillingham et al., 2008; Dietz and Stern, 2015). A growing number of IAMs incorporate some form of induced innovation (e.g., Acemoglu et al., 2012; Baldwin et al., 2020), illustrating themes that similarly emerge from our more stylized analysis. Whereas inertia in itself introduces adjustment costs, investments associated with induced innovation may be associated with cheaper enduring abatement.

Together, inertia and induced innovation together contribute to the third feature: *path dependency* in emitting systems (Aghion et al., 2016, 2019). Specifically, the enduring impact of greater abatement in one period can be found not only in induced cost reductions in specific targeted technologies, but also in changes to the overall system that yield long-term emissions reductions beyond the directly amortized costs. These may range from lasting low-carbon infrastructure (e.g., in buildings, transport, and electricity networks) and targeted low-carbon innovation to enduring changes in networks, institutions, and policy landscapes and expectations. The optimal effort involved to shift the emissions pathway in part reflects the degree of path dependency of the system.

In pursuit of transparency and tractability, our model seeks to represent the *implications* of these dynamic factors, rather than mimicking the *processes* themselves. The underlying structure can be characterized in terms of Gillingham and Stock’s (2018) distinction between static and dynamic costs. Static costs are those for which the cost of a

given degree of abatement (relative to a reference emission projection) is predetermined by exogenous modeling assumptions, conventionally represented in terms of marginal abatement cost curves. Dynamic costs are those which are incurred at a given point time in time, but which do not endure and may relate *directly* to abatement cost curves in other periods. Specifically, we introduce a term to represent rate-dependent “transitional costs” comprising two components: a *characteristic transition time* of the emitting system, and a parameter representing the contribution of transitional investments to reducing the “static” component of abatement costs. We characterize the latter as the *pliability* of the system—the extent to which, ultimately, the system can endogenously adjust to abatement.

Our analytic solution illustrates the influence of key parameters, and also reveals qualitatively different behaviors, with three distinct patterns of optimal emission pathways (regimes): the conventional results arising from models with static abatement costs only, one with moderate transitional costs, and one with predominantly transitional costs. A system with purely static abatement, akin to that in the standard DICE model, implies a sudden drop in annual emissions (i.e., an initial discontinuity in annual emissions). Intuitively, it makes sense to abate up to the value of avoided discounted climate damages.

The system behaves fundamentally differently in the presence of transitional costs. First, it transforms at a steadier rate, as higher inertia (i.e., longer characteristic transition times in our model) smooths the pace of reductions over time. As the initial abatement effort focuses on transforming the system, it is not immediately associated with reduced emissions (as emissions cannot suddenly drop, as in static-cost models). Nevertheless, the initial optimal abatement effort may substantially exceed that in purely static-cost models. Second, to the extent that the transitional costs are associated with reductions in enduring static costs (e.g., through induced innovation, infrastructure, and other path-dependent effects represented in our model by non-zero pliability), there are enhanced benefits to early action. The optimal effort thus exceeds the immediate social cost of carbon emissions. Third, faced with considerable inertia, it may (depending on the form of assumed damages of constraints) be optimal to “overshoot” into a period of negative emissions which draw peak temperatures downward.

The analysis offers several important contributions to the literature seeking to introduce dynamic characteristics into stylized IAMs. Our introduction of a representative characteristic transition time of the emitting system, indexing a cost associated with *accelerating change* from the reference emissions trajectory, offers a proxy for the wide range of specific capital stock and technology growth-rate assumptions required in more complex models. It thus offers a simple approach to evaluating the importance of the resulting inertia.

Similarly, the introduction of a parameter characterizing the pliability of the emitting system reflects the implications of extensive research that a significant fraction of the cost of abatement—classically presented as exogenous assumptions extending far into the future—may in reality be transitional. We thereby capture the broad implications of low-carbon learning-by-doing, scale economies, and infrastructure investments, all of which imply lower enduring “static” cost of low carbon options relative to high-carbon ones. Consequently, our model allows us to explore what is implied if the *ex-ante* abatement costs that are typically assumed to be unavoidable in most models are in fact pliable—awaiting the required scale of investment to secure the breakthroughs needed to set energy systems on a different course.

We also provide analytic insights to results found in more complex numerical models. For example, it is well known that cost-benefit results are sensitive to the assumed discount rate r . We derive solutions from our model for the optimal degree of initial effort and find that some elements are exceptionally sensitive to the discount rate (i.e., proportional to $1/r^6$) in the “static” case, if the reference trajectory is rising significantly and there are no exogenously defined backstop technologies. We further find that any degree of pliability necessarily increases the optimal initial investment.

We also include an Appendix to demonstrate how a version of DICE extended to include transitional costs—DICE-PACE²—compares with our analytic model. The results indicate that the simplifications of our analytic framework do not invalidate the key structural findings of DICE-PACE. Moreover, the remaining differences illustrate some of our analytically derived insights into key sensitivities, and underline that the most critical questions concern how abatement costs decline over time. Specifically, the comparison highlights the standard but critical assumptions in DICE that abatement costs decrease autonomously over time, independently of prior actions, hence facilitating rapid abatement “later on”. This stands in contrast to our model’s approach that investment in abatement is required to reduce future costs, and smooth the effort, over time.

Finally, we derive analytic solutions when faced with a hard temperature constraint, representing an extreme non-linearity in damages. These show that with only static costs, the optimal solution tends to involve “overshoot” followed by steeply negative emissions as the constraint approaches. More realistic dynamic assumptions result in very different strategies, tending to linearly reduce emissions towards zero, though with potential “overshoot” when the constraint is very tight given inertia in the emitting system.

Our aim is to highlight the central importance of considering dynamic factors, which in particular, affect the initial optimal effort and trajectory, and potential long-run outcomes. Overall, we show that in the presence of dynamic factors, transitional investments reduce long-term abatement costs, curtailing the overall costs of climate change (i.e., the sum of climate damages and abatement costs). Even with moderate climate-damage assumptions, we find that the optimal long-run temperature increase may well lie below 2 degrees Celsius.

Section 2 describes the core model, and its interpretation in the context of economic growth. In Section 3 we present and discuss the analytic solution, computing it for calibrated parameter values. Section 4 analytically calculates the optimal abatement efforts at time zero, while Section 5 addresses the cost of delaying action. Section 6 summarizes some comparative studies and sensitivities, and Section 7 derives results in the context of a fixed temperature target. Section 8 discusses some policy implications, and Section 9 concludes. Appendices present details of the solutions and some results compared to insertion of transition costs in to an adapted form of the DICE model.

2. The model

2.1. High-level optimization problem

We define cumulative emissions at time t , relative to pre-industrial times, as $E(t)$ measured in gigatonnes of carbon (GtC). We take $t = 0$ to mean today. The historical path of $E(t)$, i.e., for $t \leq 0$, is fixed and cannot be changed. $E_{\text{ref}}(t)$ is a reference trajectory that matches historical cumulative emissions for $t \leq 0$. Cumulative emissions to date are fixed at $E_0 := E_{\text{ref}}(0)$.

Going forward, i.e., for time $t > 0$, $E_{\text{ref}}(t)$ represents a reference scenario absent any substantial abatement effort. This trajectory is suboptimal in the context of climate change, such that $E(t)$ will optimally diverge from $E_{\text{ref}}(t)$ for $t > 0$. For notational simplicity, annual emissions are denoted by $e(t) := E'(t)$, measured in GtC per year (GtC/yr). The reference trajectory of annual emissions is written as $e_{\text{ref}}(t) = E'_{\text{ref}}(t)$. The constant $e_{\text{ref}}(0) = e_0$ represents current-day emissions.

In its most basic form, the stylized high-level problem we are interested in solving is

$$\min_{\{E(t)\}_{t=0}^{t=T}} \int_0^T \exp(-rt) F[E(t), e(t), e'(t)] dt, \tag{1}$$

$$\text{s.t. } E(0) = E_0, \tag{2}$$

$$\text{and } e(0) = e_0, \text{ (this restriction is optional)}. \tag{3}$$

Here $\{E(t)\}_{t=0}^{t=T}$ denotes the path of cumulative emissions $E(t)$ from $t = 0$ to $t = T$, where $T > 0$ measured in years (yr) is the time horizon, which may be infinite, $r > 0$ is the discount rate, and $F[\cdot, \cdot, \cdot]$ is a function depending on $E(t)$ and its first two derivatives, denoted $e(t)$ and $e'(t)$. The cumulative emissions path $E(t)$ for $0 \leq t \leq T$ together with the boundary conditions (2) and optionally (3) implies the annual emissions path $e(t)$ for $0 \leq t \leq T$, as well as its rate of change, $e'(t)$. This means that $E(t)$ can—without loss of generality—be used as the control variable.

The function $F[\cdot, \cdot, \cdot]$, measured in trillions of USD per year, is the sum of a climate-damage function $D[\cdot]$ and an abatement-cost function $C[\cdot, \cdot]$, both with the same units:

$$F[E(t), e(t), e'(t)] := D[E(t)] + C[e(t), e'(t)]. \tag{4}$$

The damage function reflects the form in the majority of stylized IAMs, which relate climate damages to global temperature change, using the finding that this is closely proportional to cumulative CO₂ emissions (neglecting shortlived gases). Consequently, at any given point in time, climate damages $D[\cdot]$ depend on cumulative emissions up to that point, i.e., $E(t)$, and omitting lags for simplicity (see Section 2.2 for details).

Abatement costs $C[\cdot, \cdot]$, on the other hand, depend on both annual emissions $e(t)$ and their rate of change, $e'(t)$. Most stylized IAMs take $C = C[e(t)]$, i.e., without dependence on $e'(t)$, such that the abatement cost at time t depends solely on the annual emissions at time t , relative to the reference level. As indicated, these “static” costs represent the classic structural form of an abatement cost curve. To these we add transitional costs by allowing $C[\cdot, \cdot]$ to depend additionally on $e'(t)$. As outlined, this comprises the elements of inertia and induced innovation (see Section 2.3 for details).

The model is intended principally to explore the impact of this dynamic element on optimal results. Eqs. (1)–(4) have no direct representation of economic growth. In Section 2.5, however, we show how the results from the core model can be interpreted as a model with both damage $D[\cdot]$ and abatement costs $C[\cdot, \cdot]$ scaling with economic growth, with corresponding adjustment to the discount factor r through the classic Ramsey formula. The model is a simple partial equilibrium model, without direct feedback from climate damages to the underlying assumptions around economic growth. However, the applications of most cost-benefit models suggest relatively little impact of climate change on economic growth (if they did not, they would prescribe far greater efforts than reported, for example, by the DICE model). In Appendix C we compare our results against an adapted version of DICE, i.e., DICE-PACE, *inter alia* to demonstrate that assumptions around the cost structure of abatement can have far more impact on results than the presence or absence of general equilibrium interactions.

The minimization problem (1) is subject to constraint (2), implying that cumulative emissions $E(t)$ must be continuous at $t = 0$: we cannot instantly extract carbon from the atmosphere. Models with static costs, which optimize annual emissions in sequential equilibria, yield discontinuities in annual emissions when climate damages are introduced, and steep reductions if a low-carbon technology suddenly becomes competitive. With transitional dynamics, however, such jumps in global emissions are implausible (and very costly). Constraint (3) implies that $E(t)$ smoothly matches the reference trajectory at $t = 0$, by making annual emissions $e(t)$, too, continuous at $t = 0$. To maintain comparability with standard models without inertia, this constraint is optional.

² The DICE-PACE model was developed in Grubb et al. (2021b), where PACE stands for Pliable Abatement Cost Elements.

2.2. Climate-damage function

It is now well established that global temperature change is closely related to cumulative CO₂ emissions. A central estimate is that global temperatures increase by 1 degree Celsius with each additional 600 GtC in cumulative emissions (IPCC, 2021, Table SPM.2). In line with much of the stylized literature, including the common default assumption in DICE, we assume that global damages increase quadratically with temperature. The climate-damage function, $D[\cdot]$, is thus simply:

$$D[E(t)] = \hat{d} E(t)^2, \tag{5}$$

where $\hat{d} \geq 0$ is a damage parameter with dimension trillion USD/(yr × GtC²); hence, annual damages $D[E(t)]$ have the dimensions of trillion USD/yr. The relevant quantity in our analytic solution turns out to be $8 \times \hat{d}$, so for convenience we define $d := 8 \times \hat{d}$, such that the solution can be reported in terms of d (naturally, with the same dimensionality as \hat{d}); this is without loss of generality. Table 1 gives the calibrated value $d \approx 2 \times 10^{-5}$ with appropriate units; this is equivalent to a numerical value $\hat{d} := d/8 \approx 2.5 \times 10^{-6}$ in Eq. (5).

Damage function (5) ignores any time lag between emission reductions and their impact on temperature. Contrary to common assumptions, this time lag is relatively small: Ricke and Caldeira (2014) estimate the median time lag (until maximum warming occurs) to be just over 10 years (see also Mattauch et al., 2020). A lag of L years could be introduced with a simple transposition from $d/8E(t)^2$ to $d/8E(t-L)^2$. This would reduce the net present value of the damage by a factor $(1 - \exp(-Lt))$, a minor change with no impact on the structural insight of the paper. For simplicity, we omit this.

2.3. Abatement-cost function

We specify the abatement cost function, $C[\cdot, \cdot]$, in terms of abatement $a(t)$, and its rate of change $a'(t)$ as follows:

$$C[e(t), e'(t)] := c [q a(t)^2 + p \hat{\tau}^2 a'(t)^2], \tag{6}$$

where abatement $a(t) := e_{\text{ref}}(t) - e(t)$. (7)

Here $c > 0$ is an overall cost-scaling constant, measured in trillion USD × yr/GtC². Abatement at time t , $a(t)$ is measured in GtC/yr relative to baseline, while $a'(t)$, in GtC/yr², represents its rate of change. The resulting cost function expresses annual expenditure on emissions abatement in trillions of USD per year.

The first term in Eq. (6) captures the static costs, the traditional stylized formulation of abatement costs as a nonlinear function of the degree of abatement relative to a baseline projection, scaled by q where $q \in [0, 1]$ is a dimensionless number. In common with several other stylized models, we assume that the static abatement cost at time t increases quadratically with the abatement effort at time t , giving rise to a term that scales with $a(t)^2$.³

The second term captures the transitional cost, which is proportional to the square of the rate of change of abatement and measures how rapidly the system is forced to deviate from the reference trajectory.⁴ Here we introduce two parameters, $\hat{\tau}$ (with a simple scaling to τ) and p :

³ Vogt-Schilb et al. (2018) also assume quadratic abatement costs. Nordhaus (2013) has $a(t)^{2.8}$, reduced to 2.6 in DICE2016 and some others follow that DICE assumption. Grubb et al. (2018) show that learning-by-doing tends to reduce not only the scale but the convexity of the marginal cost curve.

⁴ Though there is less evidence on the functional form of transitional costs, they are clearly convex (see Grubb et al., 2018). Vogt-Schilb et al. (2018) assume that the cost of capital retirement increase quadratically with the pace and Bauer et al. (2016) assume that costs for renewable deployment increase quadratically with the rate of accelerating renewable expansion in the REMIND model.

- Parameter $\hat{\tau} > 0$, measured in years, reflects the intrinsic inertia—i.e., the resistance to change—of the system in terms of a characteristic transition time (Harmsen et al., 2021): the higher $\hat{\tau}$, the longer it takes to achieve a given level of abatement for a given cost (or the more costly it is to overcome this inertia). The characteristic time therefore offers a compact proxy for capital stock retirement and growth rate constraints on new technologies in more complex models. In the solution, the analytically convenient quantity appears to be $\tau^2 = \hat{\tau}^2/2$, so we take $\hat{\tau} = \sqrt{2}\tau$ and report our results in terms of τ ; this is without loss of generality.
- Parameter $p \in [0, 1]$ reflects the “pliability” of the system, with $q = 1 - p$ being its complement. We use p to explore the implications of some portion of costs being transitional that otherwise, in a purely exogenous framework, would have been attributed to static costs. The case $p = 0$ represents the classical case in the sense that the abatement cost is exogenously defined at each point in time. It is extreme in the sense that our model does not explicitly incorporate exogenous cost reductions. The case $p = 1$ represents the opposite extreme, in which all costs turn out to be transitional, with innovation and infrastructural developments resulting in low carbon technology systems which ultimately become fully competitive with the incumbent fossil-fuel-based industries. Indications of such possibilities include not only the extensive literature on induced innovation cited in the introduction, and the progress observed in renewable technologies. At a wider system level, it appears that many countries have, given time, adjusted to energy price shocks without increasing overall energy expenditure, due to induced changes in technologies and structure (Bashmakov et al., 2024). Recent econometric studies add to the evidence of substantial long-run changes in energy demand in response to prices (Agnolucci et al., 2024), with an important role for directed technical change (Hassler et al., 2021).

Central to our model is the contention that *ex-ante* estimates of (exogenously defined) marginal cost curves in practice implicitly conflate static and dynamic costs in two aspects. First, much innovation and cost reduction—beyond public R&D—is induced by economic incentives and associated investment, involving private R&D, iterative economies of scale, learning-by-doing, and supply chain developments associated with deployment. Second, an important element of the costs facing many new technologies arises from mismatch with existing physical and institutional infrastructures, as, for example, seen by the needs of electric vehicles for different fueling and maintenance infrastructures. These and other factors mean that introducing a carbon price (and other policies) may involve an initial cost which declines as the economy adjusts (Aghion et al., 2016; Shapiro and Metcalf, 2023).

Inertia and induced innovation contribute to path dependence.⁵ The ratio $p\tau^2/q$ can be taken to represent the degree of path dependency of the system: if there is high inertia and significant pliability, then after transitional abatement in one period, the system will have extensive path dependence and will tend to stick to its new trajectory.

2.4. Reference scenario and base case parametrization

To complete the model setup, the reference path of cumulative emissions is specified by assuming linear growth in annual emissions as follows:

$$e_{\text{ref}}(t) := e_0 + e_1 t, \quad t \geq 0, \tag{8}$$

⁵ As noted in the introduction, other factors, such as enabling infrastructure and institutional changes, also contribute to path dependence. Beyond these, path-dependencies are also obviously implicit in the wider literatures on the adaptive nature of human beings and systems, including culminations with recent treatises on adaptive preferences (von Weizsäcker, 2024) and adaptive markets (Lo and Zhang, 2024).

Table 1
Overview of symbols, dimensions, and calibration.

| Symbol | Meaning | Dimension | Calibration |
|------------------------------|--|--|----------------------|
| r | Net discount rate (see Table 2) | 1/yr | 0.02 |
| p, q | Pliability and its complement $q = 1 - p$ | none | |
| t, T | Time, time horizon | yr | |
| $\hat{\tau}$ | Characteristic transition time | yr | 21 |
| $\tau = \hat{\tau}/\sqrt{2}$ | Rescaled characteristic transition time | yr | 15 |
| $C[\cdot, \cdot], D[\cdot]$ | Abatement cost and climate damage | trillion USD/yr | |
| c | Abatement cost parameter | trillion USD/(yr \times GtC ²) | 0.026 |
| \hat{d} | Damage parameter | trillion USD/(yr \times GtC ²) | 2.5×10^{-6} |
| $d = 8\hat{d}$ | Rescaled damage parameter | trillion USD/(yr \times GtC ²) | 2×10^{-5} |
| $E(t), E_{\text{ref}}(t)$ | Cumulative emissions at time t | GtC | |
| E_0 | Cumulative emissions at $t = 0$ | GtC | 665 |
| $e(t), e_{\text{ref}}(t)$ | Annual emissions at time t | GtC/yr | |
| $a(t)$ | Abatement at time t , $a(t) := e_{\text{ref}}(t) - e(t)$ | GtC/yr | |
| e_0 | Annual emissions at $t = 0$ | GtC/yr | 10.4 |
| $e'(t), e'_{\text{ref}}(t)$ | Rate of change of annual emissions at time t | GtC/yr ² | |
| $a'(t)$ | Rate of change of abatement at time t | GtC/yr ² | |
| e_1 | Linear reference case growth of annual emissions | GtC/yr ² | 0.1 |

$$E_{\text{ref}}(t) = E_{\text{ref}}(0) + \int_0^t e_{\text{ref}}(s) ds = E_0 + e_0 t + \frac{e_1}{2} t^2, \quad t \geq 0, \quad (9)$$

where $e_0 \geq 0$ is the annual emissions at $t = 0$, while $e_1 \geq 0$, measured in GtC/yr², represents a (linear) growth rate of annual emissions assuming “business as usual”.

Parameter e_1 enables us to explore key uncertainties regarding the future scale of the abatement challenge, which in reality involves assumptions about energy demand trends and resource availability. Changing e_1 thus allows us to explore sensitivities around the extent to which assumptions on exogenous trends affect the challenge of decarbonization.

Problem (1) has now been specified in its entirety. Table 1 contains an overview of all symbols used as well as their dimensions, and where relevant calibrated parameter values (see Section 3.3).

2.5. Interpretations with economic growth

Most stylized economic cost-benefit models involve economic growth assumptions, and express damages and abatement costs as a percentage of GDP. Our basic model specified in Eqs. (1)–(9) does not explicitly do this, but it can be readily interpreted for cases of constant economic growth rates.

With an economic growth rate $g > 0$, the damage and abatement cost functions $D[\cdot]$ and $C[\cdot, \cdot]$ can be multiplied by $\exp(gt)$, adjusting the discount factor following the Ramsey rule for discounting in the context of economic growth. The Ramsey rule combines the pure rate of time preference ρ , adjusted for the growth rate g , and the elasticity of social marginal utility of consumption η , which is typically taken to represent inequality aversion.

This transforms our optimization (1) into:

$$\min_{\{E(t)\}_{t=0}^T} \int_0^T e^{-(\rho+\eta g)t} \{e^{gt} D[E(t)] + e^{gt} C[e(t), e'(t)]\} dt \quad (10)$$

The net discount rate equivalent to r in Eq. (1) hence becomes $r = \rho + (\eta - 1)g$. Table 2 indicates different combinations of the pure rate of time preference ρ , economic growth rates g , and the elasticity of marginal utility η , which span plausible assumptions yielding gross discount rates in the range 3.5%–5%. All three variants in Table 2 correspond to a net discount rate of $r = 0.02$, which is consistent with Table 1.

With current global GDP close to one trillion USD, the numerical value of annual abatement expenditures can also be interpreted as %GDP for any of the variants in Table 2. The first variant in Table 2 is used to facilitate a comparison with DICE (see Appendix C).

Table 2
Three variants implying the discount rate $r = 0.02$.

| | Variant 1 | Variant 2 | Variant 3 |
|--|-----------|-----------|-----------|
| Pure rate of time preference (P RTP) ρ | 1.0% | 0.5% | 1.5% |
| GDP annual growth rate g | 1.5% | 3.0% | 2.0% |
| Marginal utility of consumption η | 1.67 | 1.50 | 1.25 |
| Gross discount rate $\rho + \eta g$ | 3.5% | 5.0% | 4.0% |
| Net discount rate r when damages and abatement costs are scaled by GDP | 2.0% | 2.0% | 2.0% |

3. Analytic solution and primary results

3.1. Statement of Theorem 1

Optimization problem (1) permits an analytic solution as described here.

Theorem 1. Consider optimization problem (1) with damage parameter $d > 0$ and infinite time horizon ($T = \infty$). The optimal path is

$$E(t) = E_* + e_* \cdot t + \sum_{j=1}^2 Z_j \exp(z_j t/2), \quad t \geq 0, \quad (11)$$

where constants E_* , with dimension GtC, and e_* , with dimension GtC/yr, are

$$E_* = \frac{8 c q (e_0 r - e_1)}{d} + 64 e_1 r^2 \left(\frac{c p \tau^2}{4 d} - \left(\frac{c q}{d} \right)^2 \right), \quad e_* = 8 \frac{c q e_1 r}{d}. \quad (12)$$

The form of the exponential constants z_j , with dimension 1/yr, and Z_j , with dimension GtC, for $j = 1, 2$ depend on the pliability p of the system relative to a critical threshold p^* , defined as

$$p^* := 1 - \frac{\sqrt{1 + 4x} - 1}{2x} \in (0, 1),$$

where $x := c/(d\tau^2) \in (0, \infty)$ is a dimensionless characteristic of the system. The solution then comprises three distinct regimes:

1. **No pliability.** Assume $p = 0$ and impose constraint (2). Then

$$z_1 = r - \sqrt{r^2 + \frac{d}{2 c q}}, \quad Z_1 = E_0 - E_*, \quad z_2 = 0, \quad Z_2 = 0. \quad (13)$$

2. **Medium pliability.** Assume $0 < p \leq p^*$ and impose constraints (2) and (3). Then $z_1 = z_+$ and $z_2 = z_-$, where

$$z_{\pm} = r - \sqrt{v \pm \sqrt{u}} < 0, \quad (14)$$

where $u := \frac{q^2}{p^2 r^4} - \frac{d}{c p r^2}$ and $v := r^2 + \frac{q}{p r^2}$. Both z_1 and z_2 are real valued and strictly negative. The exponential constants Z_j for $j = 1, 2$ are given by

$$Z_1 = \frac{2(e_0 - e_*) + z_-(E_* - E_0)}{z_+ - z_-} \quad Z_2 = \frac{2(e_0 - e_*) + z_+(E_* - E_0)}{z_- - z_+} \tag{15}$$

3. **High pliability.** Assume $p > p^*$ and impose constraints (2) and (3). Then $z_1 = z_+$ and $z_2 = z_-$, where

$$z_{\pm} = r - w \pm i \frac{\sqrt{|u|}}{2w}, \tag{16}$$

where $i = \sqrt{-1}$, $w := \frac{\sqrt{v + \sqrt{v^2 + |u|}}}{\sqrt{2}}$, while u, v remain as under point 2. Both z_1 and z_2 are complex valued, with real parts that are strictly negative. Constants Z_1, Z_2 remain as in Eq. (15), but with z_{\pm} as in Eq. (16).

Proof. A standard application of the calculus of variations (e.g., Goldstein et al., 2013) gives a fourth-order differential equation for the solution $E(t)$. Conjecture (11) yields expressions for E_* and e_* , as well as a fourth-order polynomial equation for the constants z_j for $j = 1, 2$. Two roots can be discarded because of the (implicit) boundary condition at $T = \infty$. The two remaining roots can be found analytically, giving z_1 and z_2 . Depending on the regime, these are either real (no-pliability and medium-pliability regimes) or complex (high-pliability regime). The constants Z_j for $j = 1, 2$ follow from the boundary conditions at $t = 0$ and are expressible in terms of z_1 and z_2 . In all cases, the cumulative emissions path $E(t)$ remains real and implies the marginal emissions path $e(t)$ by taking the first derivative. Details of the proof can be found in Appendix A.

3.2. Discussion of Theorem 1

Eq. (11) in Theorem 1 gives the optimal solution $E(t)$ for $t \geq 0$. As a sanity check, it can be verified that the limits (i) $d \rightarrow 0$ or (ii) $c \rightarrow \infty$ imply $E(t) \rightarrow E_{\text{ref}}(t)$. That is, when (i) damages are zero or (ii) the abatement cost approaches infinity, the optimal path is equal to the reference path. For non-zero damages ($d > 0$) and finite abatement cost ($c < \infty$), the path of $E(t)$ lies below that of $E_{\text{ref}}(t)$ for $t \geq 0$. In particular, Eq. (11) gives $E(t)$ as the sum of a constant E_* , a linear function of time with slope e_* , and a sum of two exponential functions. The (real parts of the) exponential parameters z_j for $j = 1, 2$ are negative, such that, as $t \rightarrow \infty$, these terms vanish.

In all cases, the optimal marginal emissions path $e(t)$ is implied by the cumulative emissions path $E(t)$ via a straightforward differentiation with respect to time. Further, in all cases an analytic solution remains possible even for a finite optimization horizon T . The resulting, somewhat more involved, expressions are available from the authors upon request.

Theorem 1 implies that optimal long-run annual emissions $e(t) = E'(t)$ are constant at the level e_* given in Eq. (12). This reflects the assumed equal convexity of damages and abatement costs, i.e., emissions are determined by the balance of damages and abatement costs, both of which increase quadratically in our model if reference emissions are rising, in the absence of assuming exogenous declines in emissions intensity or abatement costs.⁶

⁶ This reflects a point which is intuitively obvious but rarely made explicit: whether or not a cost-benefit model converges to zero emissions depends on the relative convexity of assumed damage and abatement costs, unless other factors in effect reduce the convexity of the abatement cost curve. If climate damages rise with temperature (or equivalently, with assumed risk aversion) more non-linearly than abatement costs rise with the degree of abatement, emissions necessarily fall to zero. DICE assumes the opposite in its default

As might have been anticipated, e_* is an increasing function of the abatement-cost parameter c , the reference emissions growth parameter e_1 , and the discount rate r , and a decreasing function of the damage parameter d . For a fully pliable, path-dependent system ($p = 1, q = 0$), or stable reference emissions ($e_1 = 0$), we have $e_* = 0$, i.e., it is optimal in the long run to decarbonize entirely—as would be the case for more convex damages (see footnote, and as illustrated with Theorem 2 below).

The three regimes differ in how this optimal asymptotic emissions level e_* is reached. The no-pliability solution, where $p = 0$, is akin to the standard DICE solution, and implies an initial drop in emission (prompt jump in abatement).⁷ Whenever $p > 0$, i.e., for a system with any positive degree of inertia, such an immediate response is impossibly costly. Hence, in regimes 2 and 3, the path of $e(t)$ remains continuous at $t = 0$, avoiding a discontinuity in annual emissions. In the medium-pliability regime, the optimal long-run emissions level is reached by more steadily cutting emissions to this level. The abatement effort (cost) at time zero typically exceeds that in regime 1, because part of this effort is related to the transformation of the emitting systems, the results of which are not immediately visible in the marginal emissions path.

In the high-pliability regime, the exponential parameters z_j for $j = 1, 2$ are imaginary. Naturally, the cumulative emissions profile $E(t)$ remains real valued. The corresponding response can be equivalently written in terms of trigonometric functions as follows:

$$E(t) = E_* + e_* \cdot t + \exp\left(\frac{\hat{z}t}{2}\right) \left[\frac{2(e_0 - e_*) + \hat{z}(E_* - E_0)}{\tilde{z}} \sin\left(\frac{\tilde{z}t}{2}\right) + (E_0 - E_*) \cos\left(\frac{\tilde{z}t}{2}\right) \right], \tag{17}$$

where e_*, E_* remain as in Theorem 1, while \hat{z}, \tilde{z} are real numbers defined as $\hat{z} := \text{Re}(z_+)$ and $\tilde{z} := \text{Im}(z_+)$.

The intuition for the third regime is that, when accelerating abatement is expensive (but damages are significant), this constrains the pace of abatement, leading to some degree of “overshoot” of concentrations and temperature before later correcting (steering back, with negative emissions). This explains the appearance of trigonometric functions: emissions oscillate toward the long-term optimum. We note that “overshoot” has become of feature of scenarios to meet more ambitious temperature goals in many complex IAMs of the IPCC (IPCC, 2022), but has not generally been observed in stylized cost-benefit models which lack the relevant dynamics.

3.3. Calibration

This section describes how we calibrate the parameters of our model. An overview of the calibrated parameter values is given in Table 1.

Emissions. The emissions are calibrated for fossil-fuel and industry-related CO₂ emissions only, for which there are accurate data and broadly known abatement characteristics.⁸ We define $t = 0$ to be 2019 and take values for E_0, e_0 , and e_1 from the most recent IPCC

values (damages rise quadratically with temperature but abatement costs scale with power 2.6), but this is more than offset by an assumed steep exogenous decline in abatement costs; see Appendix C.

⁷ In practice, plots from DICE do not show this because emissions at time $t = 0$ are set equal to the actual emissions and the discontinuity occurs in the first unconstrained five-year period, shown as $t + 5$.

⁸ Land use emissions in the aggregate remain highly uncertain. The fact that the major non-CO₂ emissions have relatively short lifetimes mean they cannot be treated in the same way: CO₂ cumulation drives long-term climate change, though continuing emissions of short-lived gases make a significant contribution for a few decades. The omission of non-CO₂ gases in particular means that the temperature impacts in absolute terms are underrepresented in our model.

report (IPCC, 2021). Specifically we set $E_0 = 665\text{GtC}$, $e_0 = 10.4\text{GtC}$ ($38\text{GtCO}_2/\text{yr}$), and $e_1 = 0.10\text{GtC}/\text{yr}^2 = 0.367\text{GtCO}_2/\text{yr}^2$, the latter being somewhat higher than recent rates of per-capita emissions growth.

Discount rate. We take a net discount rate for Eq. (1) to be 2.0% per year. This is a compromise between “prescriptive” and “descriptive” rates, drawing on the expert elicitation survey by Drupp et al. (2018), and can be equated with various combinations of pure time preference, economic growth rates and elasticity of marginal utility, as per Table 2. After a few decades it leads to significant discounting of costs and damages.

Climate damages. Our climate-damage estimates draw upon Nordhaus (2013) and Howard and Sterner (2017), both of which present damage estimates as proportional to the square of global temperature change, as in our model. Howard and Sterner’s (2017) “preferred damage specification” is almost four times the Nordhaus (2013) value. We take a central benchmark value midway between these, resulting in $d = 0.00002$ trillion USD/(yr \times GtC²), which corresponds to $\hat{d} = 2.5 \times 10^{-6}$.

Abatement costs. The vast majority of literature specifies abatement costs in terms of marginal abatement costs, some derived for specific projected years. Based on the extensive review by Harmsen et al. (2021, Figure 1) of 17 different complex IAMs, we take an average benchmark abatement cost parameter $c = 0.026$ trillion USD \times yr/GtC², equivalent to a marginal abatement cost of 367 USD/tC = 100 USD/tCO₂ for 50% emissions reduction from reference (7GtC/yr), in the middle of their reported range.

Characteristic transition time. There is little empirical literature on transition timescales. The review by Harmsen et al. (2021) of modeling diagnostics for complex process-based IAMs introduced a metric of “inertia timescales” for the first time, finding a wide spread in the inertia timescale of the models with mean 13.5 years. However, in general these models tend to represent only capital stock lifetimes and (sometimes) growth rate constraints on new technologies. The only model specifically developed to analyze inertia (including urban forms), the IMACLIM model, reports a timescale of 20.8 years. A review of macro-level energy systems data over 50 years and more concludes that energy systems adjust (with high pliability) to price shocks, but take around quarter of a century to do so (Bashmakov et al., 2024). A recent econometric study (Agnolucci et al., 2024) with data from the past 2–4 decades, estimates an adjustment half-life (decay rate) in response to energy price changes of around 18 years. We take our benchmark value as $\hat{\tau} = 21$ years, implying $\tau = \hat{\tau}/\sqrt{2} = 15$ years.

3.4. Results for Theorem 1

We present main results from our core model for five different scenarios. Fig. 1 displays annual emissions (in GtC per year), global mean temperature increases with respect to pre-industrial times (in degrees Celsius), along with annual damages from climate change and annual abatement costs (in trillion USD per year) for a system with no pliability (i.e., $p = 0$), a system with full pliability (i.e., $p = 1$), and three scenarios that relate to regions in between (i.e., $p \in \{0.25, 0.5, 0.75\}$). As described in Section 2.5, the figures on damages and abatement costs can also be directly interpreted in terms of percentages of GDP when the model is scaled in relation to projected GDP growth.⁹ As indicated, the results in terms of temperature and damages reflect impacts from CO₂ accumulation; including short-lived gases would increase these numbers.

⁹ To be precise, the numbers in panels (c) and (d) in Fig. 1 also represent the percentage of GDP at a given point in time multiplied by 100 trillion USD.

No pliability, $p = 0$. The system with purely static costs resembles that used in DICE and other classical IAMs (hereafter, “classical”), as discussed in Section 2. There is a prompt reduction in annual emissions (by about one third), but after this initial drop, emissions continue to rise steadily throughout the century. This is because abatement in this scenario cannot keep up with the rising emissions from the reference scenario. As can be seen, behavior in the classical case with exogenous and static costs ($p = 0$) is similar to the climate-policy ramp observed in DICE. Annual abatement expenditure increases from about 650 billion USD per year, or 0.65% of GDP, to more than treble that in 2100. As cumulative emissions continue to rise, the CO₂-driven global mean temperature increase (from pre-industrial levels) exceeds 2.0 degrees Celsius by 2100, and continues rising beyond. Damages increase correspondingly over time, almost quadrupling by the end of the Century.

As soon as $p > 0$, an immediate (discontinuous) emissions reduction is no longer possible. As indicated above, emissions in these positive-pliability scenarios initially decline (approximately) linearly. Once they cross below the classical $p = 0$ case, which happens after roughly $\tau = 15$ years, the behavior varies widely across the different cases.

Full pliability, $p = 1$. At the opposite extreme, the scenario with a fully pliable system is one with solely transitional costs and no static costs. Emissions decline steadily and reach net zero emissions around 2065. Afterwards, net annual emissions become negative, implying that cumulative emissions will decrease (as indicated for the high-pliability regime). With temperature increases proportional to cumulative CO₂ emissions, the associated global mean temperature increases to about 1.6 degrees Celsius above pre-industrial levels at the time of net zero, and decreases slightly thereafter.

With full pliability, the optimal policy involves substantially higher initial expenditure than in the other scenarios. Initial annual abatement investment in the fully pliable system is about 1.5 trillion USD per year, 2.5 times greater than in the non-pliable system. The trend is also reversed: optimal effort decreases rather than increases over time, reaching less than 500 billion USD per year by 2100. Damages in this scenario remain much lower than in the other two cases: between one and two trillion USD per year between 2018 and 2100.

Intermediate pliability, $p = 0.25, p = 0.5, p = 0.75$. In all intermediate cases, emissions after the crossing point stay below those in the classical ($p = 0$) case, but do not reach zero; as explained above, they asymptote towards a constant level. Given the absence of a “backstop” technology in our model, global temperatures, damages, and abatement costs all keep rising, though the $p = 0.75$ case stays below 2 degrees Celsius, reflecting an initial doubling of the effort, which remains above the classical cost for the first few decades, reaping the rewards in the second half of the Century.

While damages are lower for higher p in panel (c), abatement expenditures in panel (d) for $p > 0$ (but $p \neq 1$) are (mostly) above the classical case with $p = 0$. The intuition is that the larger initial abatement effort in the intermediate-pliability cases leads to reduced damages later on. Panel (e) shows abatement costs as a fraction of the climate damages by year. The standard approach ($p = 0$) spends less on abatement than the damages incurred throughout (with this parameterization, little over half as much). As pliability increases, this rises to a point at which the world initially invests more in abatement than the damages being incurred (this is still without any risk-weighting for uncertainties arising in further climate change).

Hence, in terms of overall discounted costs, any $p > 0$ ultimately reduces the net present cost associated with responding to climate change, i.e., a lower value of the (optimized) objective function (1). However as we explore in Section 4, it has the opposite implication for the optimal initial effort.

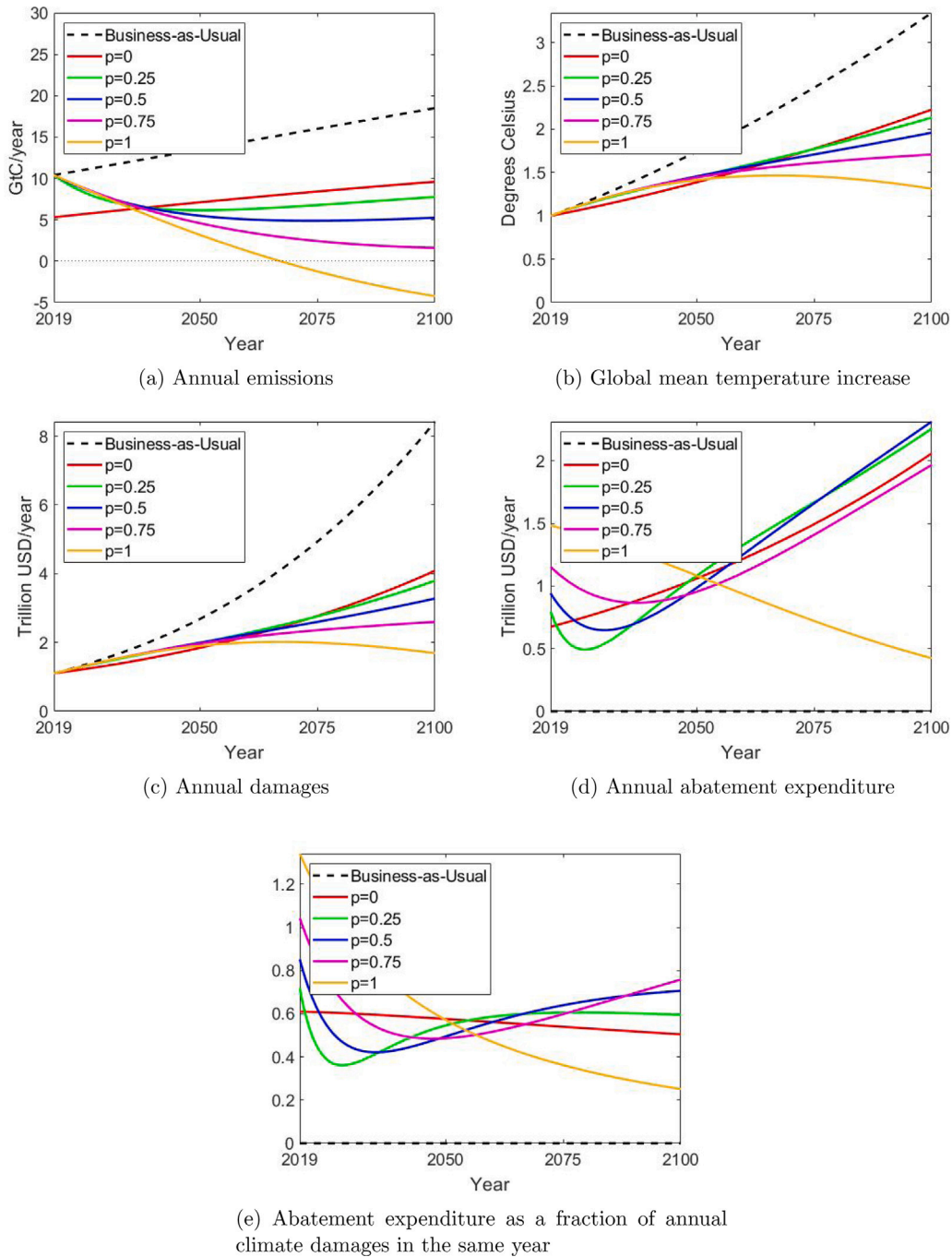


Fig. 1. Optimal policy and implications for $p \in \{0, 0.25, 0.5, 0.75, 1\}$.

4. Abatement effort at time zero

Having obtained the optimal path of cumulative emissions $E(t)$ in three regimes in Theorem 1, we can directly compute the optimal degree of initial abatement effort (expenditure at time $t = 0$), and explore key sensitivities.

4.1. No inertia, no pliability regime

For $p = 0$, substituting the exact solution (11) into the cost function $C[\cdot, \cdot]$ in Eq. (6) and evaluating the result at $t = 0$ yields the optimal current abatement effort measured in trillion USD/yr as follows:

$$C[e(0), e'(0)]|_{p=0} = c \left[e_0 - e_\star - \frac{z_1}{2} (E_0 - E_\star) \right]^2 \Big|_{p=0, q=1}, \tag{18}$$

$$= \left[\frac{e_1^2}{r^6} + \frac{2e_0e_1}{r^5} + \frac{e_0^2 + 2e_1E_0}{r^4} + \frac{2e_0E_0}{r^3} + \frac{E_0^2}{r^2} \right] \cdot \frac{d^2}{64c} + O(d^3),$$

where E_\star and z_1 are given in Eq. (12) and (13), respectively. The second line is a straightforward first-order Taylor expansion in the square of the damage parameter. From this expression, it is clear that the optimal level of effort today is extremely sensitive to the discount rate r , which appears to the power of six in the denominator whenever $e_1 \neq 0$. The ratio d^2/c confirms that effort tends to increase nonlinearly with d , while higher abatement cost c suppresses effort because it reduces the benefit/cost ratio (and with discounting it is cheaper to defer the effort required). In terms of initial emission conditions, e_0 , e_1 , and E_0 all also increase the optimal effort.

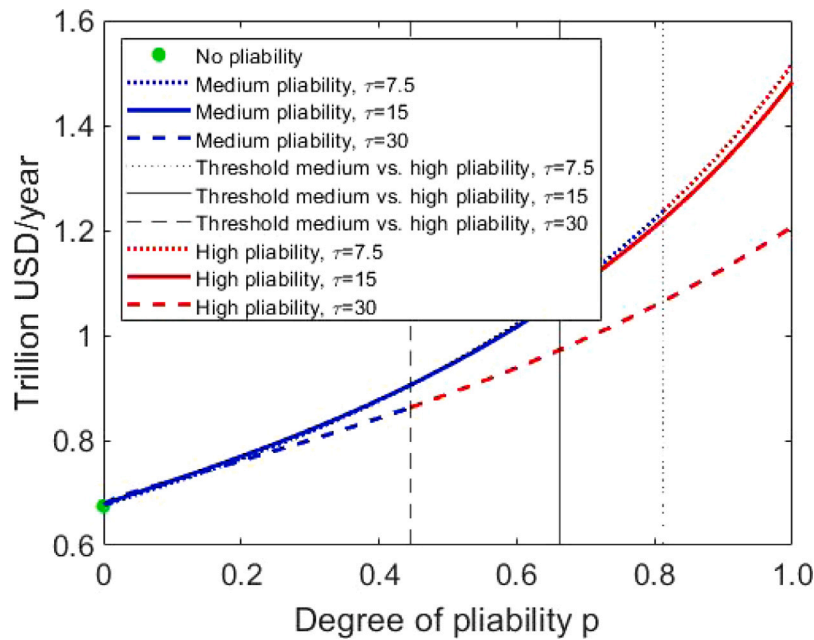


Fig. 2. Optimal abatement effort at time zero from Eq. (19).

Numerous studies with DICE have underlined sensitivity to the discount rate, but to our knowledge none have identified it analytically to such a remarkable degree. Note that the first two terms in the expansion are driven by e_1 , while the inverse quartic and cubic dependencies involve e_0 .

We interpret this as follows: without inertia, the solution suggests that in the presence of climate damages, optimal emissions today are much lower than actual emissions. At low discount rates, the solution suggests an immediate, large reduction in the starting level, which is amplified further to counteract the rising trend of future emissions. In published results from DICE and similar numeric models, this immediate reduction in annual emissions is somewhat obscured by the five-year time steps typically used, but the underlying logic is one of a sudden, potentially dramatic jump so as to “start from somewhere else”. In isolation of any consideration of dynamic constraints, it is unclear how useful this is as a policy-relevant insight, since the global energy system clearly cannot make overnight jumps in its emission levels and trajectories, as acknowledged by inclusion of a constraint on rate of change in first period in a recent implementation of DICE itself (Nordhaus, 2019).

4.2. Inertia and positive-pliability regimes

For $p \neq 0$, substituting the exact solution (11) into the cost function $C[\cdot, \cdot]$ in Eq. (6) and evaluating the result at $t = 0$ yields the optimal current abatement effort as follows:

$$C[e(0), e'(0)] = 2c p \tau^2 \left[e_1 - (e_0 - e_*) \cdot \frac{z_1 + z_2}{2} - (E_* - E_0) \cdot \frac{z_1 \times z_2}{4} \right]^2, \tag{19}$$

For a small degree of pliability (low p), the dependencies can be clarified as:

$$C[e(0), e'(0)] = C[e(0), e'(0)]|_{p=0} + V \times c \tau \sqrt{2p} + O(p), \tag{20}$$

where V is a constant,¹⁰ e_* and E_* are as in Eq. (12), while z_1 and z_2 are as in Eq. (14) (medium-pliability regime). Eq. (20) is a straightforward Taylor expansion in powers of p around the point $p = 0$, where the first term is given in Eq. (18). In Eq. (20), the fact that the second term scales with τ reflects the fact that with more inertia, greater effort is required to change the emissions trajectory, in proportion to the characteristic timescale of the emitting system. The dependence on \sqrt{p} shows that effort is very sensitive to p as p approaches 0. As soon as there is any transitional cost, i.e. for any $p > 0$, the system cannot be moved to a different starting point as in the no-pliability regime; hence, the high sensitivity to p can be explained by this qualitatively different nature of the solution. In general, the optimal initial effort increases with p , as more effort is exerted into transforming the system.¹¹

4.3. Numerical illustration

Fig. 2 displays the optimal abatement effort at time zero for our calibration from above, but with three values of the transition time τ , i.e., $\tau \in \{7.5, 15, 30\}$.

Optimal initial effort is increasing in p and decreasing in τ . The gains of induced innovation can easily be reaped in a flexible system with low inertia. If, however, inertial timescales put a serious brake on the optimal pace of abatement achieved, this dampens the response, and hence the benefits available, in the entire system. Policies to remove obstacles to faster transitions—many of which may be political and distributional—enhance the gains, and consequently, the justified effort.¹²

¹⁰ V is defined as follows

$$V = \left[e_1 - (e_0 - e_*) \frac{r + z_-}{2} + (E_0 - E_*) \frac{r z_-}{4} \right] \cdot \left[2(e_0 - e_*) - z_-(E_0 - E_*) \right] \Big|_{p=0, q=1},$$

$$z_{\pm} = r \pm \sqrt{r^2 + \frac{d}{2c}},$$

¹¹ A similar equation as Eq. (20) but for $p > p^*$ can be obtained by plugging $p = 1$ into Eq. (19). As this yields no new insights, we do not display this formula. It is available upon request.

¹² The impact of characteristic transition time τ is reversed if the climate damages are more convex: with a fixed temperature constraint, the constancy

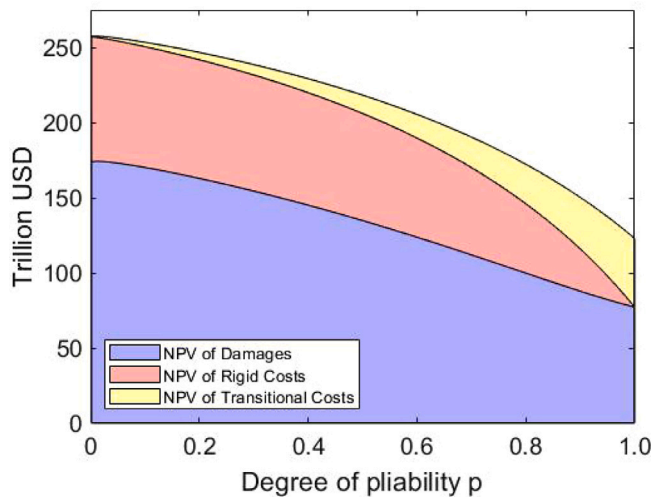


Fig. 3. Decomposition of optimal value of objective function (1).

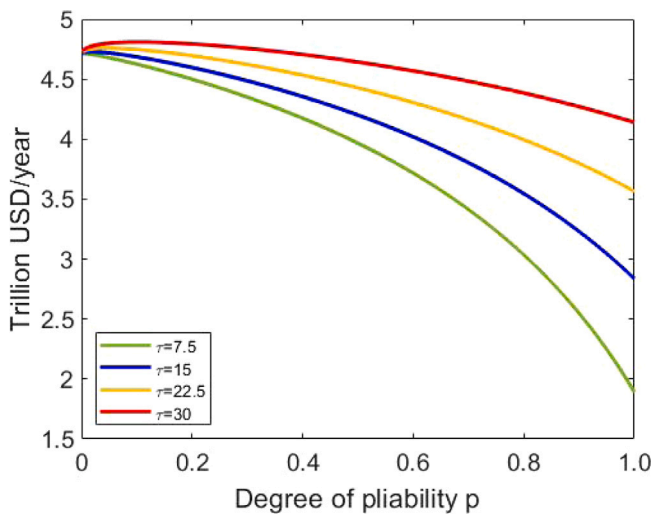


Fig. 4. Cost of delay for $\tau \in \{7.5, 15, 22.5, 30\}$.

5. Cost of delay

We explore the influence of dynamic factors on the cost of delay, which we define as the sensitivity of objective function (1) to an infinitesimal delay dt in implementing the optimal solution given in Theorem 1. During this short period of delay, the emissions profile is assumed to equal its reference trajectory, after which the re-optimized policy is implemented. Optimization problem (1) is formally unchanged after a delay dt if we recognize that the initial conditions E_0 and e_0 have shifted to $E_0 + e_0 dt$ and $e_0 + e_1 dt$, respectively.¹³

First, we analytically compute (see Appendix B) the value of the objective function (1) under the optimal policy given in Theorem 1. We refer to this quantity as the optimal net present value (NPV) associated with problem (1). Fig. 3 shows how this optimal NPV, including its three components (damages, static cost, and transitional cost), vary

of initial pace of emissions reduction implies the optimal initial effort required is proportional to τ^2 ; see Section 7.

¹³ For analytic tractability we choose an infinitesimal delay dt ; the results can be generalized to allow for any (non-infinitesimal) delay $\Delta t > 0$.

Table 3

Abatement costs (trillion USD/yr) under varying discount rates.

| PRTP rate | 0.5% | 1.0% | 1.5% |
|----------------------------|------|------|------|
| Net discount rate r | 1.5% | 2.0% | 2.5% |
| $p = 0$ | | | |
| Initial expenditure | 1.27 | 0.67 | 0.40 |
| Expenditure after 50 years | 2.28 | 1.38 | 0.90 |
| $p = 0.5$ | | | |
| Initial expenditure | 1.68 | 0.94 | 0.56 |
| Expenditure after 50 years | 2.24 | 1.50 | 1.03 |
| $p = 1$ | | | |
| Initial expenditure | 2.23 | 1.48 | 0.96 |
| Expenditure after 50 years | 0.82 | 0.80 | 0.73 |

with p . The optimal NPV is decreasing in p , while climate damages make up around two thirds of the total across the range.

Second, we analytically compute (again, see Appendix B) the sensitivity of the optimal NPV with respect to a short delay dt . The results shown in Fig. 4 demonstrate that the cost of delay is decreasing in p from around ~ 3.6 trillion USD per year (for $p = 0$) to around ~ 2.6 trillion USD per year for our benchmark value $\tau = 15$. The fact that the cost of delay is a large multiple of the optimal effort at time zero suggests that the optimized objective function is highly sensitive to the initial conditions. Even as the optimal effort at time zero is relatively modest, a short delay of this optimal effort may be exceedingly costly; indeed, much more costly than the optimal effort itself.

With low pliability, climate change is more costly overall to deal with and climate damages are substantially higher. However, the system faces no inertial barrier. The ability to drop emissions immediately is valuable in terms of the large immediate marginal impact on $E(t)$, and every year that passes without such action squanders this potential, substantially increasing long-run damages. At higher pliability, abatement effort shifts towards transitional investments with enduring benefits, but the scale of (marginal) reduced climate damages is lower because the overall scale of long-run climate change is curtailed. Higher inertia, by impeding rapid response, reduces the pace at which the system can exploit lower static costs, but increases the marginal value of the achievable emission reductions. At a characteristic transition time of $\tau = 30$ years, these two effects roughly cancel each other out and the overall cost of delay is almost independent of the degree of system pliability.

6. Validation and sensitivity checks

The dominant sensitivities explored in the climate cost-benefit literature identify critical assumptions around climate damages and the discount rate. With climate damages assumed as quadratically related to temperatures (as in DICE), we illustrated that for $p = 0$, the initial expenditure increases quadratically with the value of the damage parameter d .

Our results confirm that the optimal expenditure may be extremely sensitive to the discount rate: as $p \rightarrow 0$, some terms scale to power r^{-6} (see Eq. (18)), but the overall impact of the discount rate is much more complex. Table 3 shows how the optimal expenditure in our base case varies with pliability for three different net discount rates $r = 1.5, 2$, and 2.5% . For the standard model (for which $p = 0$), the discount rate has a large impact as traced and debated widely in the literature. We find that reducing the net effective discount rate r (i.e., after GDP) from 2.0% to 1.5% , roughly doubles the initial expenditure, whilst increasing r to 2.5% cuts the optimal expenditure by 40% . Higher discount rates defer more effort to decades later. Sensitivity to the discount rate declines somewhat as pliability increases.

On the surface, our broad conclusions about pliability lead to very different conclusions from the most established cost-benefit models, with much higher initial effort (especially at higher discount rates), but

the results in principle should be broadly comparable, at least within the constraints of the highly stylized and partial-equilibrium nature of our model.

To explore this, Appendix C compares our analytical results to an adapted version of the DICE model, DICE-PACE, extended to include transitional costs in its cost function, so as to be able to represent and compare the impact of pliability.

When assumptions on reference emissions and abatement cost parameters are aligned, the results align closely (to within about 5%). Across a range of cases, the optimal initial expenditure in the two models align to within about 25% across a range of cases for higher levels of pliability. However, they differ far more for low levels of pliability, and in particular, for the classical case of $p = 0$. Sensitivity analyses suggests that this is not because of the remaining differences in, e.g., functional forms, the complexity of the carbon cycle-temperature modules, or the difference between simple partial vs. general equilibrium treatments.

The reason can be traced to the second line of Eq. (18), which shows that the initial expenditure is inversely related to the cost of abatement, i.e., higher c implies lower initial expenditure. Economically, this is because more expensive abatement reduces the cost-benefit ratio of action, so the optimal solution is to do less. And the parametrization of DICE2016 has its initial abatement costs at least twice as high as our assumptions based on the review by Harmsen et al. (2021) review of IPCC data and models. This greatly deters initial action.

DICE tackles abatement through two core assumptions that exogenously and substantially reduce abatement costs over time. The first concerns scale of assumed costless reduction in carbon intensity of the global economy over time. The second is the autonomous reduction in abatement cost itself (“backstop cost”). The two are also multiplicative. In the standard DICE2016 parametrization, the intensity declines by 1.5%/yr, and the cost declines by 2.5%/yr. Therefore, in addition to the declining carbon intensity, the cost of abatement falls at 4%/yr, without any investment in the model. The lack of inertia also means that once abatement becomes cheap relative to rising damages, emissions can drop precipitously—curtailing climate damages, and hence the initial shadow-price of carbon. To a large degree, DICE “solves” the climate problem with an initial drop in emissions following by waiting for abatement to get cheaper (and damages, larger).

Our model represents the opposite extreme: reduction in abatement cost only occurs through effort. This is represented by positive pliability, and the lower costs associated with higher pliability, whilst inertia also constrains the ability of the model to defer action and then rapidly drop emissions.

As reviewed in our introduction, some other models have delved into some of these aspects. Our findings echo the broad conclusion of Golosov et al. (2014) that the costs of inaction are particularly sensitive to the assumptions regarding the substitutability of different energy sources and technological progress. It is increasingly recognized that dynamic and uncertainty effects justify greater up-front effort (Kalkuhl et al., 2012 and Bertram et al., 2015), including accelerated international diffusion (Schultes et al., 2018), and strengthen optimal initial effort in cost-benefit models (Baldwin et al., 2020, Grubb et al., 2021b). Campiglio et al. (2022) consider both capital stock and induced innovation and conclude that when “putting these factors together, [they] estimate a net premium of 33% on the optimal carbon price today relative to a ‘straw man’ model with perfect capital mobility, fixed abatement costs and no uncertainty”.

Quantitatively, however, our model suggests a much bigger impact than most of these studies (if pliability is high). This is because, rather than focusing on independent components, it applies these concepts to the overall emitting system.

7. Results with fixed temperature target

We end by considering another key outstanding assumption: the convexity of damages, or more prosaically, the risk of unacceptably

high damages and risks associated with a tipping point threshold in the interactions of the earth’s climate with human systems.

A key structural assumption in the preceding analysis concerns the functional form of damages, in particular that damages for a given temperature increase with the same (quadratic) form as abatement costs. If there is no reduction in abatement costs without effort (i.e. no exogenous cost reductions), this results in the specific form of the solution involving the convergence on a constant emissions rate. This reflects an ongoing trade-off between the abatement costs and climate damages, with no climate risks beyond the costs expressed as rising with the square of temperature change, which is a basic assumption in the default DICE model, although few scientists accept this.

Science has instead largely framed the problem of climate change in terms of planetary risks, and emphasized that the risk of major climate damages arising from earth systems (or indeed, some vulnerable social systems) crossing various thresholds increase sharply with the degree of temperature change.¹⁴ While solutions involving other functional forms of damages in general are not analytically tractable, we here investigate the implications of highly non-linear damages by the proxy of a limit on temperature, which can be represented by a limit on cumulative emissions. This results in our second Theorem.

7.1. Statement of Theorem 2

Theorem 2. Consider optimization problem (1) without damages (i.e., $d = 0$) but a constraint on temperature change, represented by limit on cumulative emissions E_T within a finite time horizon $T < \infty$. Let $\Delta_T := E(T) - E_{\text{ref}}(T) < 0$ and $\delta_T := e(T) - e_{\text{ref}}(T) < 0$ be desired deviations from the reference trajectory at the terminal time point (both negative).

1. **No inertia/pliability.** Assume $p = 0$. Impose initial constraint (2) and terminal constraint $E(T) = E_T$. Then the optimal path is

$$E(t) = E_{\text{ref}}(t) + \Delta_T \frac{\exp(rt) - 1}{\exp(rT) - 1}, \quad 0 \leq t \leq T. \tag{21}$$

2. **Positive inertia/pliability.** Assume $p < 0 < 1$. Impose initial constraints (2)–(3) and terminal constraints $E(T) = E_T$ and $e(T) = e_T$. Then the optimal path is

$$E(t) = E_{\text{ref}}(t) + \sum_{i=1}^2 \left[Z_i \frac{e^{z_i t} - 1}{z_i} \right] - (Z_1 + Z_2) \frac{e^{r t} - 1}{r}, \quad 0 \leq t \leq T, \tag{22}$$

where constants z_i for $i = 1, 2$ with dimensions yr^{-1} are $z_1 = (r - \sqrt{r^2 + 2q/(pr^2)})/2$ and $z_2 = (r + \sqrt{r^2 + 2q/(pr^2)})/2$. Constants Z_i for $i = 1, 2$ measured in GtC/yr are

$$Z_1 = \frac{(e^{z_2 T} - e^{r T}) \Delta_T - \left(\frac{e^{z_2 T} - 1}{z_2} - \frac{e^{r T} - 1}{r} \right) \delta_T}{(e^{z_2 T} - e^{r T}) \left(\frac{e^{z_1 T} - 1}{z_1} - \frac{e^{r T} - 1}{r} \right) - (e^{z_1 T} - e^{r T}) \left(\frac{e^{z_2 T} - 1}{z_2} - \frac{e^{r T} - 1}{r} \right)}, \tag{23}$$

$$Z_2 = \frac{-(e^{z_1 T} - e^{r T}) \Delta_T + \left(\frac{e^{z_1 T} - 1}{z_1} - \frac{e^{r T} - 1}{r} \right) \delta_T}{(e^{z_2 T} - e^{r T}) \left(\frac{e^{z_1 T} - 1}{z_1} - \frac{e^{r T} - 1}{r} \right) - (e^{z_1 T} - e^{r T}) \left(\frac{e^{z_2 T} - 1}{z_2} - \frac{e^{r T} - 1}{r} \right)}. \tag{24}$$

3. **Full pliability.** Assume $p = 1$. Impose initial constraints (2)–(3) and terminal constraints $E(T) = E_T$ and $e(T) = e_T$. Then the

¹⁴ Weitzman (2009) underlined the importance of highly non-linear risks, which he explored in terms of damages rising with temperatures with much higher degrees of non-linearity. Note that in its default assumptions, DICE2016 has abatement costs rising more non-linearly (exponent 2.6) than damages (exponent 2.0); emissions fall to zero, rather than rising *ad infinitum*, due to the assumed exogenous decline of abatement costs over time.

optimal path is

$$E(t) = E_{\text{ref}}(t) + Z_1 t + Z_2 t e^{rt} - (Z_1 + Z_2) \frac{e^{rt} - 1}{r}, \quad 0 \leq t \leq T, \tag{25}$$

where constants Z_i for $i = 1, 2$ with dimensions GtC/yr are

$$Z_1 = \frac{(\delta_T - r\Delta_T)rT e^{rT} - \delta_T(e^{rT} - 1)}{(e^{rT} - 1)^2 - r^2 T^2 e^{rT}}, \quad Z_2 = \frac{(\delta_T - r\Delta_T)(e^{rT} - 1) - \delta_T rT}{(e^{rT} - 1)^2 - r^2 T^2 e^{rT}}. \tag{26}$$

Proof. The proof is similar to that of [Theorem 1](#), requiring us to solve a fourth-order differential equation for $E(t)$. The key difference is that terminal emissions conditions are enforced at $T < \infty$. Details are available on request.

7.2. Discussion of [Theorem 2](#)

With no inertia, i.e., $\tau = 0$, initial and terminal constraints on emission rates are irrelevant since emissions can jump with no transitional cost. Behavior is then very different from the other cases. Note that with $p = 0$

$$e(t) = e_{\text{ref}} + \Delta_T \frac{r \exp(rt)}{\exp(rT) - 1}$$

When T , the time horizon, is moderately close relative to the impact of discounting, the result is a large discontinuous initial jump of magnitude $\Delta_T \frac{r \exp(rt)}{\exp(rT) - 1}$ at time 0. With a larger T , the denominator increases and the emissions trajectory can defer subsequent action, with “overshoot” followed by accelerating abatement towards large negative emissions in the final stages, at high but short-lived annual costs, extensively discounted.

With positive pliability, i.e. $p > 0$, inertia prevents the sudden jump and the dynamics are different. Time derivatives of all the terms in [\(22\)](#) involving Z_1 and Z_2 have the property of linear dependence on t when t is small.

The general equation is greatly simplified for the case of full pliability ($p = 1, q = 0$). Note that in this case we have

$$e(t) = e_{\text{ref}}(t) + Z_1 (1 - e^{rt}) + Z_2 r t e^{rt}$$

The consequence is that whereas in the cost-benefit case higher inertia leads to (slightly) less effort, with more convex damage the opposite is the case. With a temperature constraint, the optimal initial effort increases directly in proportion to τ^2 , as well as (of course) the severity of the constraint.

7.3. Results for [Theorem 2](#)

[Fig. 5](#) plots annual emissions and annual abatement costs resulting from a temperature constraint (limit on cumulative emissions), i.e., [Theorem 2](#). We set $T = 81$ and $e(T) = 0$, so that annual emissions are constrained to reach zero at the end of the century. Furthermore, $E(T) = 1000$ GtC, which equates to *remaining* cumulative emissions of 335GtC (1230GtCO₂), giving a near two-thirds chance of staying “below 2 degrees Celsius” ([IPCC, 2021](#)).

For the classical case of $p = 0$, we see the expected immediate jump in annual emissions. Thereafter, emissions, however, do not start to decline further until the middle of the century. Shortly after 2090 they go below zero, into a period of negative emissions—at costs approaching 10% of global GDP (panel (b))—in order to bring the global temperature back down. Sensitivity studies underline that this behavior is amplified considerably with higher discount rates, as without inertia, the model defers action to a steeper decline with more negative emissions to compensate for higher earlier emissions.

With positive inertia and pliability, the figure shows that emissions decline almost linearly irrespective of the degree of inertia or pliability

across the range shown, leveling out around zero in the last decade. In the context of highly non-linear damages, therefore, even modest amounts of pliability and inertia have a major impact on the optimal trajectory.

The impact of pliability and inertia on costs is amplified in this context, as indicated in panel (b). The initial expenditure is higher with significant pliability/inertia (though scarcely visible, increasing from about 0.4 to 0.7 trillion USD/yr as p goes from 0→1). The reduced abatement costs from earlier effort serves to cap the escalating costs, including for large-scale negative emissions, as the limit is approached.

Summarizing, with a fixed binding goal for temperature (and hence cumulative emissions), the behavior of the system has parallels with the cost-benefit case when the system has no learning or inertia—an initial jump, followed by deferral of stronger action, until the constraint approaches (or as damages accumulate). With $p > 0$, however, the emissions trend inclines towards linear reductions in the earlier stages, to minimize transitional costs and exploit the possibilities of lowered abatement costs.

8. Policy implications

Our model emphasizes the importance of understanding dynamic characteristics of emission reductions and the energy transition. We have shown that dynamic factors, which we represent in a stylized and aggregate form in terms of pliability and transition times, amplify the optimal present effort under a cost-benefit assessment and generally lead to lower long-run temperature change, climate damages, and overall costs. In the context of a fixed temperature goal, the optimal effort rises sharply with inertia (represented by the transition time) to secure an approximately linear trend towards the goal. This stands in sharp contrast to the behavior of models without such dynamics.

One implication is that optimal carbon prices should be above the social cost of carbon, a result, which, as noted in [Section 6](#), is also found in emerging literature with more complex models. In more applied terms, in addition to driving substitution, a stronger carbon price would reflect learning spillovers associated with learning-by-doing, and deter investment in carbon-intensive stock which would risk becoming “stranded assets”.

However the policy implication of our aggregated, systems-level analysis logically goes beyond this. Carbon emitting stock has varied lifetimes (as an example, one can compare lights, vehicles, power plants, and buildings). Different abatement technologies may also have different feasible growth rates. Therefore different components embody different degrees of inertia (i.e., different transition times). This suggests that it may be optimal to focus initial efforts on sectors with higher inertia, at least when faced with a constraint on temperature and/or cumulative emissions. This view is supported by, for example, [Vogt-Schilb et al. \(2018\)](#). Their findings are summarized in their title: “when starting with the most expensive option makes sense”. [Baldwin et al. \(2020\)](#) also find that “if dirty capital cannot be converted to other capital, then it is optimal to stop investing in dirty capital earlier (compared to a case in which investment is reversible)”.

Clean technologies also differ in, for example, their scope for innovation and learning-by-doing. Thus, the potential spillovers from deployment and the scale of potential benefits varies between options. Policies such as those introduced to drive solar PV deployment were expensive per unit of emission reduction, but [Newbery \(2018\)](#) concludes they were a cost-effective way of fostering a technology revolution that now facilitates much cheaper global decarbonization.

The policy implication therefore is not just a higher carbon price.¹⁵ The implication is that the optimal response involves a variety of actions. Other modeling approaches drawing on dynamic systems and

¹⁵ As [Acemoglu et al. \(2016\)](#) observe, “relying only on carbon taxes or delaying intervention has significant welfare costs”.

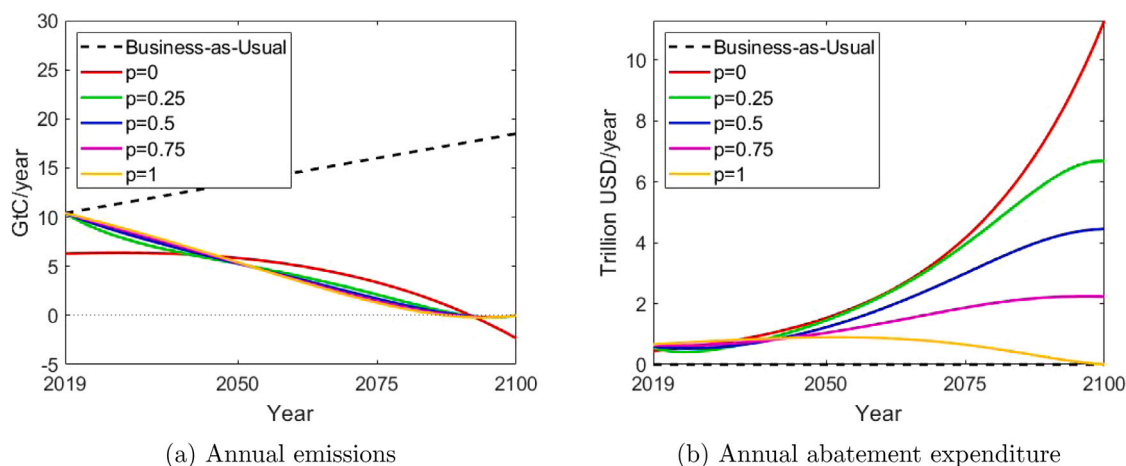


Fig. 5. Optimal policy and implications with limit on cumulative emissions.

agent-based approaches (e.g., Dosi and Nelson, 2010; Lamperti et al., 2018) also point to the need for multiple instruments to tackle these distinct dimensions.

A carbon price reflecting estimates of the social cost of carbon—if politically feasible—could of course help provide a bedrock. However in the context of inertia, induced innovation, and the path dependencies of the real-world systems that drive emissions, an optimal approach has to complement this with additional and more targeted efforts to drive an efficient energy transition. The transitions literature has in recent years increasingly focused on need for and design of policy packages (Rogge et al., 2017), whilst Grubb et al. (2023) articulate the economic case for “three pillars” of policy strategies for low carbon transitions.

Consequently, the heightened investment we identify as optimal indicates an overall effort that needs to be deployed across multiple different instruments, so as to minimize carbon-intensive lock-in, enhance innovation throughout technological systems, and facilitate more rapid transition to low carbon pathways.

9. Conclusion

We have constructed a stylized model that focuses on dynamic features of abatement costs, splitting the latter into static costs and elements of transitional costs. Whilst most emerging efforts in this area involve growing complexity of process-based model structures and intensive data requirements, our model in contrast is deliberately stripped down, to a limited number of essential components, introducing aggregate representative concepts. The aim has been to transparently illuminate the potential impact of including dynamic factors in a stylized way that also enables analytic solutions.

Our results illuminate key sensitivities, and demonstrate that the relative degree (as well as the absolute scale) of these cost components has important implications for optimal trajectories, initial effort and policy design, and long-run economics of the system.

Capital stock and other factors create aggregate inertia, represented in our model in terms of a characteristic transition time $\hat{\tau}$. Since technology deployment is typically associated with cost reductions, and investments in energy-related infrastructure (e.g., buildings, electricity and fuel/charging networks) may last many decades, we argue that most *ex-ante* estimates of marginal cost curves in practice conflate static and transitional cost elements. The extent to which technologies and systems may adjust to transitional abatement-related effort is characterized in terms of the *pliability* of the system.

Many models assume marginal abatement costs to be exogenously definable, necessarily positive, and enduring subject to assumptions of exogenous decline or “backstop” technologies—assumptions which

tend to defer stronger action. Compared to this classical formulation, the optimal response in systems with high pliability tend to start with optimally linear reductions in emissions, driven by higher initial effort, and result in lower long-run temperature change and damages. In a cost-benefit setting, higher (lower) inertia slightly dampens (amplifies) these benefits in our model. In a setting with highly convex damages—represented in extreme by a fixed temperature goal—inertia is critical, with the optimal effort in our model increasing with $\hat{\tau}^2$.

The extent to which *ex-ante* assumed (static) abatement costs are actually transitional is of course uncertain. However, we note that, for example, numerous modeling studies from a decade ago assumed *ex-ante* that solar energy and electric vehicles would be enduringly expensive ways of reducing emissions. Initially, they were, but with hindsight it is clear that much of the early investment in deployment drove innovation and scale economies. Hence, we contend that many technology cost projections in reality conflate assumed static costs with the need for transitional investment, or adjustment costs, at technology or system levels.

Moreover, whilst zero technology cost is impossible, abatement is not a technology but a difference between the cost of low and high carbon technologies. Over the past decade, following large investments, several low carbon technologies have indeed become as cheap as fossil fuels, suggesting that exploring a full range of $0 \leq p \leq 1$ is an important research inquiry. We have shown how much it matters.

Our aim is that this model will inspire further research on the dynamic features of emitting systems. Gaining a deeper understanding of different approaches to dealing with inertial timescales, induced innovation, and path dependency is crucial to help inform policymaking on one of the most important threats facing humanity this century.

CRedit authorship contribution statement

Michael Grubb: Conceptualization, Data curation, Formal analysis, Methodology, Project administration, Validation, Writing – original draft, Writing – review & editing. **Rutger-Jan Lange:** Formal analysis, Methodology, Writing – original draft, Writing – review & editing. **Nicolas Cerkez:** Formal analysis, Resources, Software, Visualization, Writing – review & editing. **Ida Sognaes:** Validation, Writing – original draft. **Claudia Wieners:** Conceptualization, Data curation, Formal analysis, Methodology, Software, Validation. **Pablo Salas:** Conceptualization, Investigation.

Data availability

Data will be made available on request.

Appendix A. Proof of Theorem 1

Euler–Lagrange equations. The optimization problem described in (1) can be solved using standard Euler–Lagrange (EL) methods. The only non-standard feature is that the control variable $E(t)$ appears alongside both of its first and second derivative in the integrand F . For this reason, the standard EL equation is adjusted to include a third term as follows

$$0 = \frac{d(e^{-rt}F)}{dE} - \frac{d}{dt} \frac{d(e^{-rt}F)}{dE'} + \frac{d^2}{dt^2} \frac{d(e^{-rt}F)}{dE''}. \tag{A.1}$$

where $F := F(E, E', E'')$ as in Eq. (4), where primes denote derivatives. Explicitly computing all derivatives we obtain

$$\begin{bmatrix} 1 \\ E(t) \\ E^{(1)}(t) \\ E^{(2)}(t) \\ E^{(3)}(t) \\ E^{(4)}(t) \end{bmatrix} \begin{bmatrix} -4p\tau^2 e_1 r^2 - 2q(e_0 r + e_1(r t - 1)), \frac{d}{dc}, 2qr, 4p\tau^2 r^2 - 2q, -8r p \tau^2, 4p\tau^2 \end{bmatrix} = 0,$$

which we here express as an inner product involving $E(t)$ and its four derivatives. This expression makes clear that in general we are faced with an inhomogenous linear ordinary differential equation (ODE) of fourth order. As is standard, the solution can be written as the sum of two solutions: one that solves the homogenous ODE and one that solves the inhomogenous ODE.

Solution to inhomogenous ODE. The inhomogenous ODE can be solved by a linear function of time, which we write as

$$E(t) = E_\star + e_\star t, \tag{A.2}$$

where E_\star and e_\star are constants to be found. For this candidate solution $E(t)$, the second, third, and fourth derivatives are zero. Solving the simplified ODE for B and b , we obtain

$$e_\star = 8 \frac{c q e_1 r}{d} \quad E_\star = \frac{c q (e_0 r - e_1)}{d/8} + 64 e_1 r^2 \left(\frac{c p \tau^2}{4 d} - \left(\frac{c q}{d} \right)^2 \right). \tag{A.3}$$

This simple solution already yields one important insight into the long-term behavior of our solution: in the long run, optimal cumulative emissions are linear in time, such that annual emissions are optimally constant. Specifically, the optimal long-run constant level of emissions is given by the parameter e_\star above. As can be seen, it decreases with the damage parameter d , but increases with the static-cost component q , the discount rate r , and the increase of marginal emissions in the reference scenario, given by e_1 .

While the inhomogenous ODE determines the optimal long-term emissions path, the particular solutions to the homogeneous ODE determine the optimal course of action in the short-term. We discuss these next.

Solution to homogenous ODE. To solve the homogenous ODE, we look for solutions that are exponential in time. Indeed, the homogenous ODE of fourth order allows for four independent solutions taking the form

$$E(t) = \sum_{j=1}^4 Z_j \exp\left(\frac{z_j t}{2}\right), \tag{A.4}$$

where the parameters Z_j and z_j remain to be determined for $j = 1, 2, 3, 4$. Substituting this candidate solution into the ODE and simplifying, we find that the constants z_j for each $j = 1, 2, 3, 4$ must solve the following fourth-order polynomial equation

$$\left[d/c, 4 q r, 4 r^2 p \tau^2 - 2 q, -4 r p, p \tau^2 \right] \begin{bmatrix} 1 \\ z_j \\ z_j^2 \\ z_j^3 \\ z_j^4 \end{bmatrix} = 0, \quad j = 1, 2, 3, 4. \tag{A.5}$$

Generally, this equation is of fourth order, unless $p = 0$, in which case it is only of second order (note the last two entries of the row vector).

Full solution. The full solution is obtained by summing the solutions to the homogeneous and inhomogeneous ODEs, i.e.,

$$E(t) = E_\star + e_\star t + \sum_{j=1}^4 Z_j \exp\left(\frac{z_j t}{2}\right), \tag{A.6}$$

where the parameters E_\star and e_\star are given by (A.3), the constants z_j for $j = 1, 2, 3, 4$ are the roots of the fourth-order polynomial equation given in (A.5), and the four constants Z_j for $j = 1, 2, 3, 4$ remain to be determined by four boundary conditions, as discussed below. These boundary conditions will need to ensure that $E(0) = E_\star + \sum_{j=1}^4 Z_j = E_0$, thereby putting a constraint on the Z_j 's.

Boundary conditions. In general, the four constants Z_j are determined by a total of four boundary conditions to be specified at either $t = 0$ or $t = T$. At $t = 0$, we impose $E(0) = E_0$, reflecting the fact that cumulative emissions (relative to pre-industrial times) at time zero are fixed. For systems with any positive transitional cost ($p > 0$), we also impose $E'(0) = E'_{\text{ref}}(0) = e_{\text{ref}}(0) = e_0$, because sudden jumps in marginal emissions would incur an infinite cost. By imposing both boundary conditions, we ensure that the path of cumulative emissions $E(t)$ smoothly matches that of the reference trajectory of cumulative emissions $E_{\text{ref}}(t)$.

At $t = T$, we are faced with two free boundary conditions, as endpoint $E(T)$ and its derivative $E'(T)$ are left to be determined by the optimizer. However, in the limit as $T \rightarrow \infty$, which we consider below, two of the four homogenous solutions can be discarded (set to zero), as they blow up exponentially, thereby causing infinite damages. As such, only two constants Z_j , for $j = 1, 2$ remain, which can be determined by the two boundary conditions at $t = 0$.

If $p = 0$, the ODE and polynomial equation are of second order. In this case, only a single boundary condition at $t = 0$ is required, which we take to be $E(0) = E_0$. In this case, a jump in marginal (but not cumulative) emissions at time zero is permitted.

Solutions under three regimes. The optimal solution behaves differently, qualitatively, depending on the numerical values of the parameters. Specifically, three regimes can be identified. We present the solution in each of three mutually exclusive and collectively exhaustive regimes:

1. No pliability: $p = 0$,
2. Medium pliability: $0 < p \leq p_\star$, which implies $c q^2 \geq p \tau^2 d$,
3. High pliability: $p > p_\star$, which implies $c q^2 < p \tau^2 d$.

The critical boundary between the medium- and high-pliability regimes is denoted p^\star and is determined by setting p equal to p^\star , q equal to $1 - p^\star$ and solving for p^\star the equality $c q^2 = p \tau^2 d$, i.e., we must solve

$$c(1 - p^\star)^2 = p^\star \tau^2 d.$$

This is a quadratic equation in p^\star with two potential solutions. Only one of these potential solutions falls in the range $(0, 1)$, which reads

$$p^\star := 1 - \frac{\sqrt{1 + 4x} - 1}{2x} \in (0, 1),$$

where $x := c/(d\tau^2) \in (0, \infty)$ is a dimensionless characteristic of the system. For $0 < p \leq p^\star$, it can be verified that $c q^2 \geq p \tau^2 d$, such that we are in the medium-pliability regime. For $p > p^\star$, we can be verified that $c q^2 < p \tau^2 d$, such that we are in the high-pliability regime.

In each case, an analytic solution is possible, which can be found by (i) solving the (in general) fourth-order polynomial equation, (ii) discarding two of the four solutions to the homogeneous ODE that correspond to the explosive solutions, and (iii) imposing the relevant boundary condition(s) at $t = 0$. We here only report the analytic solution in the case where $T = \infty$, which is economically the most relevant, and for which the solution takes the simplest possible form.

Zero pliability: If $p = 0$, such that the system contains no pliability, the fourth-order ODE simplifies to a second-order ODE. The corresponding second-order polynomial equation allows for two unique roots, one positive and one negative. The positive root can be discarded as it corresponds to an explosive solution, such that we can set $Z_2 = Z_3 = Z_4 = 0$, leaving only Z_1 to be determined. The negative root is given by

$$z_1 = r - \sqrt{r^2 + \frac{d}{2cq}}. \tag{A.7}$$

Note that $z_1 < 0$; the other root contains a plus instead of a minus in front of the square root and is economically irrelevant. This confirms the first part of Eq. (13) in Theorem 1. Imposing the boundary conditions $E(0) = E_0$, the constant Z_1 can be determined as

$$Z_1 = E_0 - E_*, \tag{A.8}$$

where the value of E_* is given by (A.3) when p is set to zero. This confirms the second part of Eq. (13) in Theorem 1. For the zero pliability regime, we do not impose $E'(0) = e_0$ such that the optimal level of today's emissions, $E'(0)$, will generally differ from the reference level, e_0 . For pliable systems in the two regimes below, a jump in marginal emissions is impossible.

Medium pliability: If $p \neq 0$, $cq^2 \geq p\tau^2 d$, such that pliability is non-zero but small in relative terms (i.e., $0 < p \leq p^*$), the fourth-order polynomial allows for four distinct roots. Two roots are positive and can be discarded from economic arguments, i.e., we set $Z_3 = Z_4 = 0$. The two remaining (negative) roots are given by

$$z_1 = r - \sqrt{r^2 + \frac{q}{p\tau^2} + \sqrt{\left(\frac{q}{p\tau^2}\right)^2 - \frac{d}{cp}}}, \tag{A.9}$$

$$z_2 = r - \sqrt{r^2 + \frac{q}{p\tau^2} - \sqrt{\left(\frac{q}{p\tau^2}\right)^2 - \frac{d}{cp}}}, \tag{A.10}$$

where each displayed square root is a real number because $cq^2 \geq p\tau^2 d$ by assumption in the current regime, which implies $(q/p\tau^2)^2 \geq d/(cp)$. These equations confirm Eqs. (14) in Theorem 1.

Imposing the boundary conditions $E(0) = E_0$ and $E'(0) = e_0$, we find the two constants Z_1 and Z_2 as follows

$$Z_1 = \frac{2(e_0 - e_*) + z_2(E_* - E_0)}{z_1 - z_2} \quad Z_2 = \frac{2(e_0 - e_*) + z_1(E_* - E_0)}{z_2 - e_1}. \tag{A.11}$$

These equations confirm Eqs. (15) in Theorem 1.

High pliability: If $p \neq 0$, $cq^2 < p\tau^2 d$, such that transitional costs are large in relative terms (i.e., $p > p^*$), the fourth-order polynomial equation allows for four distinct, complex-valued, roots. To avoid the emissions path exploding as $t \rightarrow \infty$, we pick the two roots with negative real parts. Hence, we may set $Z_3 = Z_4 = 0$. The two negative roots z_1 and z_2 differ by only a single sign, such that we can denote them by $z_1 = z_+$ and $z_2 = z_-$, where z_{\pm} is defined as

$$z_{\pm} \equiv r - \frac{1}{\sqrt{2}} \sqrt{r^2 + \frac{q}{p\tau^2} + \sqrt{\frac{d}{cp\tau^2} - \left(\frac{q}{p\tau^2}\right)^2 + \left(r^2 + \frac{q}{p\tau^2}\right)^2}} \pm \frac{i}{\sqrt{2}} \frac{\sqrt{\frac{d}{cp\tau^2} - \left(\frac{q}{p\tau^2}\right)^2}}{\sqrt{r^2 + \frac{q}{p\tau^2} + \sqrt{\frac{d}{cp} - \left(\frac{q}{p\tau^2}\right)^2 + \left(r^2 + \frac{q}{p\tau^2}\right)^2}}}, \tag{A.12}$$

where $i = \sqrt{-1}$ is the imaginary unit, and every displayed square root is a real (positive) number, because $cq^2 < p\tau^2 d$ in the current regime. It is clear that both z_{\pm} have negative real parts as desired. This confirms equation (16) in Theorem 1.

Imposing the boundary conditions $E(0) = E_0$ and $E'(0) = e_0$, we find that the constants Z_1 and Z_2 are identical in form to those in the

medium-pliability regime, namely

$$Z_1 = \frac{2(e_0 - e_*) + z_2(E_* - E_0)}{z_1 - z_2} \quad Z_2 = \frac{2(e_0 - e_*) + z_1(E_* - E_0)}{z_2 - z_1}. \tag{A.13}$$

However, the numerical values of these constants differ from those in the medium pliability regime, because the two roots z_1 and z_2 , which appear in the numerator and denominator, are now complex values. Hence, Z_1 and Z_2 are also complex valued. Naturally, the cumulative emissions path $E(t)$ for all time t remains real valued. After some tedious but straightforward trigonometric algebra, the optimal cumulative emissions trajectory $E(t)$ can be rewritten in trigonometric terms as

$$E(t) = E_* + e_* t + \exp\left(\frac{\hat{z}t}{2}\right) \left[\frac{2(e_0 - e_*) + \hat{z}(E_* - E_0)}{\tilde{z}} \sin\left(\frac{\tilde{z}t}{2}\right) + (E_0 - E_*) \cos\left(\frac{\tilde{z}t}{2}\right) \right],$$

where e_* and E_* are as in (A.3), while \hat{z} and \tilde{z} are real numbers coming from the real and imaginary parts of z_1 above. Explicitly, we have

$$\hat{z} = r - \frac{1}{\sqrt{2}} \sqrt{r^2 + \frac{q}{p\tau^2} + \sqrt{\frac{d}{cp\tau^2} - \left(\frac{q}{p\tau^2}\right)^2 + \left(r^2 + \frac{q}{p\tau^2}\right)^2}} \tag{A.14}$$

and

$$\tilde{z} = \frac{1}{\sqrt{2}} \frac{\sqrt{\frac{d}{cp\tau^2} - \left(\frac{q}{p\tau^2}\right)^2}}{\sqrt{r^2 + \frac{q}{p\tau^2} + \sqrt{\frac{d}{cp\tau^2} - \left(\frac{q}{p\tau^2}\right)^2 + \left(r^2 + \frac{q}{p\tau^2}\right)^2}}}. \tag{A.15}$$

The intuition for the high pliability regime is that, when “steering” is expensive, it might be beneficial to “oversteer” before correcting (steering back) later, which explains the appearance of trigonometric functions in the solution: emissions oscillate towards the long-term optimum. For a fully pliable system in which case $q = 0$, it is optimal to decarbonize the economy completely at some finite time, and even go into negative marginal emissions (capturing carbon dioxide from the atmosphere), also at some finite time, while oscillating (with exponentially decreasing amplitudes) towards a fully decarbonized limit.

In all three regimes, the optimal marginal emissions path $E'(t)$ is implied by the optimal cumulative emissions path $E(t)$ via a straightforward differentiation with respect to time. Further, in all cases an analytic solution remains possible even for a finite optimization horizon T , but the resulting expressions are more involved, because it no longer holds that two out of four roots from the fourth-order polynomial can be discarded (all four roots are relevant in this case). The resulting expressions are available from the authors upon request.

Appendix B. Analytic solution for NPV and cost of delay

Assume $0 < p \leq p^*$, such that the medium-pliability regime applies; below we extend the results to all $p \in [0, 1]$. Assume $T = \infty$, i.e., an infinite time horizon. Assume the optimal cumulative emissions path $E(t)$ given in Eq. (11) in Theorem 1. Then the net present value (NPV) of damages can be computed analytically as

$$\int_0^\infty \exp(-rt) \frac{d}{8} E(t)^2 dt = \frac{d}{8} \left[\frac{2e_*^2 + 2e_* E_* r + E_*^2 r^2}{r^3} + \frac{4Z_1 Z_2}{2r - z_1 - z_2} + \sum_{i=1}^2 \left\{ \frac{Z_i^2}{r - z_i} + \frac{8Z_i(e_* + E_* r)}{(2r - z_i)^2} - \frac{4z_i Z_i E_*}{(2r - z_i)^2} \right\} \right]. \tag{B.1}$$

Second, the NPV of the static-cost component can be computed in closed form as

$$\int_0^\infty \exp(-rt) c q [e_{\text{ref}}(t) - e(t)]^2 dt = \tag{B.2}$$

$$\frac{c q}{4} \left[\frac{8e_1^2}{r^3} + \frac{4(e_* - e_0)^2}{r} + 8e_1 \frac{e_0 - e_*}{r^2} + \frac{4z_1 z_2 Z_1 Z_2}{2r - z_1 - z_2} + \sum_{i=1}^2 \left\{ 8(e_* - e_0) \frac{z_i Z_i}{2r - z_i} + \frac{z_i^2 Z_i^2}{r - z_i} - 8e_1 \frac{2z_i Z_i}{(2r - z_i)^2} \right\} \right].$$

Third, the NPV of the transitional-cost component reads

$$\int_0^\infty \exp(-rt) 2 c p \tau^2 [e'_{\text{ref}}(t) - e'(t)]^2 dt = \tag{B.3}$$

$$\frac{c p \tau^2}{8} \left[\frac{16e_1^2}{r} + \frac{z_1^4 Z_1^2}{r - z_1} + \frac{z_2^4 Z_2^2}{r - z_2} + \frac{4z_1^2 z_2^2 Z_1 Z_2}{2r - z_1 - z_2} - 16e_1 \left(\frac{z_1^2 Z_1}{2r - z_1} + \frac{z_2^2 Z_2}{2r - z_2} \right) \right].$$

In Eqs. (B.1), (B.2) and (B.3), the quantities e_* , E_* are given in Eq. (12), while z_i for $i = 1, 2$ are given in Eq. (14), and Z_i for $i = 1, 2$ are given in Eq. (15).

By adding the right-hand side (RHS) of Eqs. (B.1), (B.2) and (B.3), we obtain the optimal NPV of the entire minimization problem (1), i.e.,

$$\text{NPV} = \text{RHS of equations (B.1), (B.2) and (B.3)}. \tag{B.4}$$

This optimal NPV remains valid in the limit where p approaches zero, such that the NPV in the no-piability regime can be obtained as a special case. Moreover, all expressions technically remain valid in the high-piability regime; while some quantities turn complex, the imaginary parts cancel out and the result is a real-valued number that equals the desired NPV in the high-piability regime.

The cost of delay discussed in the main text is obtained by comparing the NPV as computed above with the NPV evaluated a small time dt later, assuming no action is taken in the meanwhile. Our solution remains valid after some delay if we recognize that the initial conditions have shifted. In particular, cumulative emissions have increased from E_0 to $E(0 + dt) = E_0 + e_0 dt$, while annual emissions have increased from e_0 to $e(0 + dt) = e_0 + e_1 dt$. Hence, with obvious notation,

$$\text{cost of delay} = \frac{d \text{NPV}}{d E_0} e_0 + \frac{d \text{NPV}}{d e_0} e_1, \tag{B.5}$$

where the NPV is given in Eq. (B.4). The cost of delay is measured in units of currency per units of time. The required derivatives can be computed in closed form by using Eqs. (B.1), (B.2) and (B.3), which depend explicitly on E_0 and e_0 . Moreover, the chain rule must be employed to account for the implicit dependence of E_* , Z_1 and Z_2 on the initial conditions E_0 and e_0 ; the resulting (lengthy) expression for Eq. (B.5) is available from the authors on request.

Appendix C. Comparison with DICE and sensitivity analysis

C.1. The DICE-PACE model and parameter choices

To explore comparability of our simple partial-equilibrium model with the far more complex structure of the DICE model, and consider some additional sensitivities, we translate the main base case assumptions of Table 1 into a version of DICE with a cost-function that is extended to incorporate transitional costs, termed DICE-PACE (see Grubb et al., 2021b). DICE has a standard growth model approach in which gross GDP depends on capital, (exogenous) labor, and the (exogenous) total factor productivity (TFP). Capital is reduced by depreciation and increased by investments, which are a fixed fraction (25%) of GDP. GDP is diminished by climate damages and abatement costs:

$$Y(t) = Y^0(t) \times (1 - D(t) - C(t)),$$

where $Y(t)$ is GDP after climate-related costs, $Y^0(t)$ is GDP, and $D(t)$ and $C(t)$ are respectively the climate damages and abatement costs, both in percentages of GDP. This means that the impacts of climate damages and abatement costs are scaled in proportion to GDP. Damages depend on global warming T (relative to pre-industrial times) as $D = k_D T^2$, where $k_D > 0$ is a damage parameter.

In the original DICE model, abatement costs are

$$C(t) = C_0(1 - \delta_C)^t \times \sigma(t) \times (\mu(t))^\beta.$$

Here, $\mu(t)$ is the abatement measured as a fraction of emissions avoided, $\sigma(t)$ the carbon intensity, i.e., default emissions per dollar GDP, C_0 , δ_C , and β are parameters (with DICE2016 parameters as summarized in Kellett et al. (2019)), and time t is measured in years. The carbon intensity is assumed to decrease exogenously with an initial decay rate of δ_σ .

The DICE cost function assumes that abatement costs decline with time, thanks to a presumed exogenous technological progress, and with carbon intensity (assuming that if there are less emissions, it should be easier to prevent or remove them). In addition, it assumes that the costs increase more than linearly with the abatement fraction ($\beta > 1$), as the first few percent of abatement should be cheaper to achieve than the last.

DICE has no representation of inertia or innovation induced by emissions abatement, or other investment. By neglecting the transitional character of at least part of the abatement costs it implicitly assumes zero pliability. However, as illustrated in Grubb et al. (2021b), it is straightforward to take pliability into account by generalizing the DICE cost curve:

$$C(t) = C_0 (1 - \delta_C)^t \times \sigma(t) \times \left[(1 - p)(\mu(t))^\beta + p \hat{\tau}^\beta \left(\frac{d\mu(t)}{dt} \right)^\beta \right],$$

where we set $\beta = 2$ for consistency with the analytical model, the pliability $p \in [0, 1]$, and the characteristic time $\hat{\tau}$ is as in the main text of the current article. This modification of DICE allows us to explore how the results from our analytic model compare against the behavior of the much more complex DICE, and to identify some of key aspects and parameter choices which may drive differences.

C.1.1. Welfare, economic growth and discounting

DICE assumes population to grow by about 20% before plateauing. As our analytical model does not include population explicitly, we set population to be constant in DICE-PACE. The TFP was tuned such that the GDP growth over the first 160 years equals 1.5%/year in the absence of climate damage and abatement cost ($C = 0, D = 0$). After 160 years, the growth rate is reduced to 0.5%/year to prevent numerical instability in the optimization. See Section 2.5 for the interpretation of our model in terms of GDP growth and discounting parameters.¹⁶ Together with the rate of pure time preference of 1.0%/year, this yields an overall discount rate of 3.5%, which applied to damage costs and abatement expenditure scaled in proportion to the GDP growth, results in the 2% net discount rate relevant to main optimization problem (1).¹⁷

C.1.2. Climate model and climate damage

As in Grubb et al. (2021b), we use the climate model DICE2016 for temperature and the DICE2013 CO₂ model, given the problems later acknowledged with the DICE2016 carbon cycle model. The climate damage parameter k_D is chosen to be three times as high as the (very

¹⁶ Our model treatment is equivalent to having GDP-weightings on these components with a higher discount rate. Our base case is for example equivalent to a 1.5%/yr GDP (per capita) growth with 3.5% discount rate and elasticity of marginal utility of 1.65, all well within the range of economic debate. Alternative combinations that would yield exactly the same result are indicated in Table 2. For example, the implemented discount rate would be identical for GDP growth at 2%/yr and pure rate of time preference 1.5%/yr with an elasticity of marginal utility at 1.25.

¹⁷ We take a constant population so that all GDP scaling results can be equated with per capita GDP, given the starting GDP. The projected population growth in DICE, which levels out after a few decades, in fact has only modest impact on overall results compared to most other factors.

Table C.1
Parameter choices in DICE and DICE-PACE.

| Quantity | DICE | DICE-PACE |
|-------------------------------------|---|--|
| δ_σ | 1.52%/year | CASES 1A+1B: 0.5%/year ($e_1 = 0.1\text{GtC/yr}$) CASES 2A+2B: 1.52%/year ($e_1 = 0.0\text{GtC/yr}$). |
| C_0 | DICE2016 default 7.4%, i.e., full (100%) abatement in first year would cost 7.4% of GDP | CASES 1A+2A: as in DICE. CASES 1B+2B: tuned to calibration from (Harmsen et al., 2021) |
| δ_C | 2.5%/year | 0.5%/year |
| β | 2.6 | 2 |
| k_D | 0.00236/degree ² | 0.0071/degree ² |
| Pure rate of time preference (PRTP) | 1.5%/year | 1.0%/year |
| Elasticity of marginal utility | 1.45 | 1.65 |
| Total Factor Productivity (TFP) | Exogenous increase (gradually declining growth rate) | Tuned to yield prescribed GDP growth (1.5%/year) |
| Population | Exogenous increase to ceiling level | Constant |

low) standard value in DICE. For the analytic model, the value in our Table 1 is calibrated to be midway between the DICE2016 default damage function and the “preferred damage specification” of Howard and Sterner (2017). With damages scaling with GDP, this approximately aligns the damage parametrization for the latter half of the century.

C.1.3. CO₂ emissions and abatement

DICE’s default for the initial (exogenous) reduction of the carbon intensity, δ_σ , is 1.52%/year. Combined with GDP growth at 1.5%/yr, this results in virtually constant reference (no-policy) emissions—consistent with no emissions growth (i.e. $e_1 = 0$) in our model. In order to also compare DICE-PACE with our main scenario of rising emissions, which features an increase in reference emissions of approximately 50% over 50 years, we also run DICE-PACE with a weaker decline in carbon intensity, $\delta_\sigma=0.5\%/yr$, which yields similar reference emissions.

With default DICE2016 parametrization, the initial cost (i.e., before exogenous reductions over time) of total abatement to zero emissions would be 7.4% of GDP. The higher non-linearity of the DICE default abatement cost curve with an exponent of 2.6 creates complications for comparison with our quadratic formulation. We explore two cases with the much lower abatement cost parameter approximating the abatement cost curve calibration in our analytic model.¹⁸

DICE has an exogenous decline in abatement cost, δ_C , of 2.5%/year. This cost reduction occurs “for free”, without any investment in the model. For comparison with the case in which cost reductions occur principally as a result of investment, we choose a modest exogenous decline of 0.5%/year.

Note that high, but strongly declining, abatement costs in DICE imply that it is optimal to keep initial abatement efforts low—in the default parameters, the combination of the two exogenous trends in cost and intensity (δ_C and δ_σ) result in an exogenous cost reduction at 4%/yr, so that unit abatement costs halve over 20 years and fall to a quarter after 40 years.

Noting that with 1.5% GDP growth the DICE reference case has almost constant emissions, and given the wide difference in abatement cost profiles between DICE2016 and our model calibration from Harmsen et al. (2021), we study four DICE-PACE simulations which differ in the following parameters, as outlined in the parameter Table C.1:

- CASE 1A: High initial abatement costs (DICE2016 C_0 parameter) and increasing reference emissions

- CASE 1B: Low initial abatement costs (our c calibration, Table 1) and increasing reference emissions
- CASE 2A: High initial abatement costs and constant reference emissions
- CASE 2B: Low initial abatement costs and constant reference emissions

All other parameters are maintained as in DICE2016.

C.2. Results

The most directly policy-relevant output is the indication of optimal abatement effort, i.e., abatement cost in DICE, which with pliability (DICE-PACE) can be translated as the scale of expenditure or effort, at a given time. The “present expenditure” (at $t = 0$) is the most immediately relevant. We also show expenditure after 50 years to give a compact view of the relationships between action and cost now and action and cost later. In Table C.2, we compare DICE-PACE and our analytical model for rising reference emissions for CASES 1A and 1B. Fig. C.1 presents results for CASE 1A.

The most striking comparative result is that, qualitatively at least, the effect of pliability is similar between both models. Initially, high pliability leads to substantially higher initial abatement expenditure, because early abatement has large benefits for the future. However, after 50 years, the abatement expenditure is smallest for very high pliability, despite higher abatement, because there are no enduring costs. There are, however, quantitative differences between CASE 1A and the analytical model:

- With the “classic case” of $p = 0$ there appears still a large difference in initial expenditure. Our model reports $2.5 \times$ the expenditure of DICE-PACE. However, there is convergence after 50 years, with the DICE-PACE expenditure rising more than five-fold relative to GDP, to 1.4% of GDP, whilst the relative expenditure in the analytic model barely doubles.
- Initial expenditures are far more closely aligned with full pliability ($p = 1$), differing by about 20% in the “A” cases. Compared to zero pliability, the expenditure in the analytic model somewhat more than doubles, whereas in DICE-PACE the effort is more than quadrupled.
- With these default settings, annual expenditures are similar after 50 years up to pliability of around 0.75. However they diverge greatly a $p \rightarrow 1$.

These underlying patterns are not radically different for the case of constant reference emissions (CASES 2A, 2B), as illustrated in Table C.3. Note the exceptionally close agreement (within 5%) across the range of pliability between the analytic model and CASE 2B, i.e., cases where the reference emissions are both constant at current levels and initial abatement costs are approximately aligned. Along with validating the analytic model, this suggests that the general equilibrium and other complexities and differences in DICE are insignificant

¹⁸ With the default DICE2016 parametrizations ($C_0 = 7.4\%$, $\beta = 2.6$), the cost of a 50% cut (before exogenous reductions over time) would be almost twice that implied in our analytic model calibration (Table 1), and the higher non-linearity means that the cost of prompt full (100%) abatement is 7.4% of GDP, almost three times higher. We adopt a “high initial cost” case with the original DICE parameter, but quadratically increasing costs (i.e., same cost as the default in DICE for full abatement, but higher costs for partial abatement). The “low initial cost” case corresponds to our cost parameter $c = 0.026$ (or 2.6% of GDP).

Table C.2
Comparative results for CASES 1A and 1B.

| Pliability | Abatement cost at $t = 0$ (%GDP) | | | Abatement cost at $t = 50$ (%GDP) | | |
|------------|----------------------------------|---------|---------|-----------------------------------|---------|---------|
| | Analytic model | CASE 1A | CASE 1B | Analytic model | CASE 1A | CASE 1B |
| 0.00 | 0.67 | 0.27 | 0.54 | 1.38 | 1.40 | 1.96 |
| 0.25 | 0.79 | 0.30 | 0.59 | 1.54 | 1.61 | 1.47 |
| 0.50 | 0.94 | 0.44 | 0.79 | 1.50 | 1.77 | 0.98 |
| 0.75 | 1.15 | 0.72 | 1.01 | 1.28 | 1.45 | 0.49 |
| 1.00 | 1.48 | 1.19 | 1.25 | 0.80 | 0.14 | 0.00 |

Table C.3
Comparative results for CASES 2A and 2B.

| Pliability | Abatement cost at $t = 0$ (%GDP) | | | Abatement cost at $t = 50$ (%GDP) | | |
|------------|----------------------------------|---------|---------|-----------------------------------|---------|---------|
| | Analytic model | CASE 2A | CASE 2B | Analytic model | CASE 2A | CASE 2B |
| 0.00 | 0.50 | 0.24 | 0.52 | 0.78 | 0.85 | 1.20 |
| 0.25 | 0.57 | 0.26 | 0.55 | 0.89 | 0.99 | 0.90 |
| 0.50 | 0.67 | 0.36 | 0.70 | 0.86 | 1.09 | 0.60 |
| 0.75 | 0.82 | 0.56 | 0.88 | 0.71 | 0.90 | 0.30 |
| 1.00 | 1.05 | 0.88 | 1.07 | 0.36 | 0.15 | 0.00 |

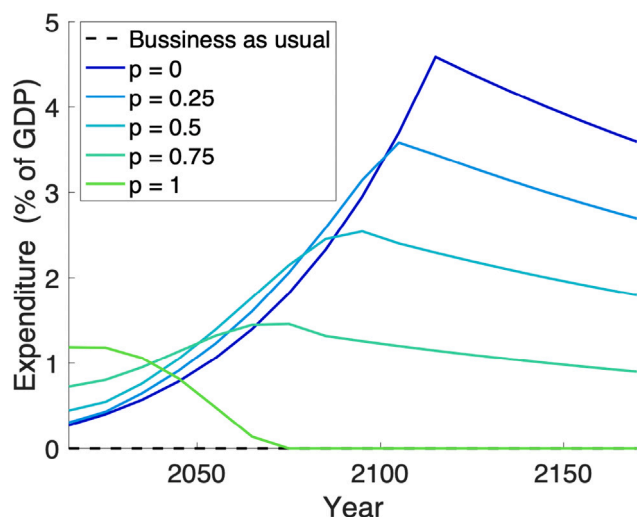


Fig. C.1. Abatement expenditure over time in DICE-PACE for CASE 1A.

compared to the underlying assumptions around abatement cost levels and dynamics, and the reference case.

The main reason for the large difference for $p = 0$ in CASES 1A and 2A turns out to be DICE-PACE’s high value for the initial abatement cost, to which DICE-PACE is highly sensitive. In CASE 1B, where the initial abatement cost parameter is reduced by a factor of almost 3, initial abatement expenditure doubles for $p = 0$, but hardly changes for $p = 1$, which aligns much more closely with the analytic results.

It may at first seem counter-intuitive that initial abatement expenditure increases if initial abatement costs decline, as we observe especially for low pliability. However, this result agrees qualitatively with our equation (18) for the optimal effort for $p = 0$ (the traditional case), the optimal abatement expenditure is (to first order) inversely proportional to the unit abatement cost. That is quite simply because a higher unit cost reduces the cost-benefit ratio of abatement, if there are no other benefits of abatement.

High abatement costs then discourage strong abatement. However, in the case of high pliability, high early investment brings down future costs of abatement, and remains comparatively more attractive despite the high initial cost. Under a lower C_0 , a similar amount of abatement expenditure for $p = 1$ buys a higher rate of change of abatement, so that in CASE 2B, full abatement is achieved (and abatement costs reach zero) within 50 years in the DICE-PACE model. In contrast, low pliability means that (for all four cases) abatement costs keep increasing

while abatement itself increases, peak when full abatement is reached, and decrease only gradually in line with exogenous cost reductions.

Combined with the assumed trend of exogenous cost reductions, and autonomous reductions in intensity which reduce the cost further, abatement becomes much cheaper over time. As long as emissions are still high, combined with rising damages, the abatement expenditure scales up dramatically to cut emissions. If combined with pliability, moreover, the higher earlier investment in brings down both emissions and costs further. Along with accumulating climate damages, after 50 years, the cost-benefit ratio of action becomes massively positive and emissions finally drop towards zero, which—along with the path-dependency implied by high pliability—means that abatement costs do too.

C.3. Conclusion

The fact that the numbers are of the same order of magnitude, and the initial efforts align well when the initial abatement and reference case assumptions are aligned, validates the consistency (and comparability) of these models.

As a numerical model, DICE allows exploration of a large range of variations, and it turns out varying assumptions around exponents, e.g., on the shape of the abatement cost function, have relatively limited impact. The far more sophisticated general equilibrium treatment in DICE, and its far more complex treatment of carbon cycle, also appear to have relatively little bearing on the results.

Alongside the traditional focus on climate damages and discount rates, what actually matters are the in-built assumptions around abatement costs. DICE assumes that the progress arrives “for free”—and that falling abatement costs meet rising damagers, after which—without any inertia—emissions can plummet. Our model—or, the corresponding adjustments in DICE-PACE—reflects the extent to which such progress in reality requires substantial investment to change the course of the energy system.

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